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A Decision Model for Tackling Logistic OptimizationProblem in Online Business Environment

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Abstract. Online business has increased during the COVID-19 pandemic, but the emergence of a number of problems, namely reduced material supply, price fluctuations because an item is difficult to distribute and slow delivery due to transportation of goods based on the type of transportation used (Trucks, Trains, Airplanes and Ships). A number of declines due to the COVID-19 virus pandemic, resulting in longer order waiting times. Pick-up and Delivery Issues are variations of Vehicle Routing Issues that appearin many real-world transportation scenarios, such as product delivery and courier services. This work studies the Pickup and Delivery Problem with Time Windows, where goods must be transported from one location to another, with taking into account certain time limits and vehicle capacity. This aims to minimize the number of vehicles used, as well as operational costs for all routes. To solve this problem, a mathematical model in the form of is used Mixed Integer Linear Programming (MILP) from Pickupand Delivery Problems with Time Windows.

Keywords: Mixed Integer Linear Programming, Pick up and Delivery

INTRODUCTION

Transportation and mobility in modern societies are seen as major concerns by people, companies and the public services. These problems have been studied with much interest by the scientific community for years, contributing toa better logistic network, cost reduction, and the improvement of service quality and urban mobility (Toth & Vigo, 2002; Van der Bruggen et al., 1993). In the field of combinatorial optimization and operations research, the problem related to transportation and mobility that has received a lot of attention is the Vehicle Routing Problem (VRP). The VRP aims at building a set of vehicle routes toattend a set of customers, so that operational costs are minimized. It is used to model several real world situations, andhas many variations, each one considering different constraints and scenarios (Savelsbergh, 1992).

This work focuses on the case where goods should be transported from one location to another, while respecting the capacity of the vehicles, as well as thespecific periods of time when goods can be picked up and delivered at each location. In the scientific literature this problem is modeled as a variation of the VRP, known as Pickup and Delivery Problem with Time Windows (PDPTW),which has a great applicability in the transportation field (Bent & Van Hentenryck, 2006; Cordeau et al., 2008). The Pickup and Delivery Problem with Time Windows is part of a wider class of problems, the so-called Vehicle Routing Problems. The VRP has more than fifty years of scientific studies, with the first work dating from the end

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of the 1950s (Dantzig & Ramser, 1959). In the VRP, there is a set of requests, or customers, with demands to be supplied by a fleet of vehicles located in a common location, the depot. The goal is to build a route for each vehicle so that all requests are attended and that costs are minimized. The definition of how a route is constructed and costs are minimized are both linked to the variant considered, more specifically, to its restrictions (Laporte, 1992; Solomon, 1987). The VRP generalizes the classical N P-Hard Travelling Salesman Problem (TSP), so itis N P-Hard as well. In fact, the TSP can be thought as a special case of the VRP, where the requests are the locations, or cities, which should be visited only once, and there is only one route that starts and ends at the same location. The cost function to be minimized is the total distance travelled. However, the TSP is usually not classified as a VRP variation, having its own set of variations. The classical and most studied variation of VRP is actually the Capacitated Vehicle Routing Problem (CVRP) (Merz & Huhse, 2008).

In it, just as in the TSP, all requests should be visited only once, but they have a certain demand, while all the vehicles have a maximum capacity to attend all demands. This capacity should never be exceeded during a route. All vehicles start and end their routes at a single common depot, and a vehicle can have at most one route. The cost function to be minimized is the total cost of all routes.

Further, another commonly studied VRP variation is the Vehicle Routing Problem with Time Windows (VRPTW). In this case, the requests have a defined time interval in which service can occur. Other restrictions are usually very close to the ones of the CVRP, and in fact most studies of the VRPTW actually consider the Capacitated Vehicle Routing Problem with Time Windows (CVRPTW), being a generalization of the CVRP. The two most common cost functions to be minimized are the total cost of the routes, and the total travel time by all vehicles. The last variation to be referred and later detailed is the Pickup and Delivery Problem with Time Windows, which generalizes the VRPTW. In the PDPTW, the vehicles no longer deliver goods from a depot to the customers, but instead the customers need goods to be transported from a pickup location to a delivery location. These visits should also respect a time interval to happen at each location, just as in the VRPTW. The cost function to be minimized is usually the total cost of all routes, the number of vehicles used, or even a combination of both (Agra et al., 2012; Kang & Lee, 2018).

LITERATUR REVIEW

Vehicle Routing Problem

The Pickup and Delivery Problem with Time Windows is part of a wider class of problems, the so-called Vehicle Routing Problems. The VRP has more than fifty years of scientific studies, with the first work dating from the end ofthe 1950s (Dantzig & Ramser, 1959). In the VRP, there is a set of requests, or customers, with demands to be supplied by a fleet of vehicles located in a common location, the depot. The goal is to build a route for each vehicle so that all requests are attended and that costs are minimized. The definition of how a route is constructed and costs are minimized are both linked to the variant considered, more specifically, to its restrictions. The VRP generalizes the classical N P-Hard Travelling Salesman Problem (TSP), so it is N P-Hard as well. In fact, the TSP can be thought as a special case of the VRP, where the requests are the locations, or cities, which should be visited only once, and there is only one route that starts and ends at the same location. The cost function to be minimized is the total distance travelled. However, the TSP is usually not classified as a VRP variation, having its own set of variations. The classical and most studied variation of VRP is actually the Capacitated Vehicle Routing Problem (CVRP). In it, just as in the TSP, all requests should be visited only once, but they have a certain demand, while all the vehicles have a maximum capacity to attend all demands. This capacity should never be exceeded during a route. All vehicles start and end their routes at a single common depot, and a vehicle can have at most one route. The cost function to be minimized is the total cost of all routes.

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METHODS

In the studied Pickup and Delivery Problem with Time Windows, a set of routes has to be constructed in order to satisfy transportation requests. The transportation requests specify an origin location, referred as pickup, and a destination location, referred as delivery. A delivery may only happen after its corresponding pickup (so called precedence constraint). A fleet of identical vehicles is available to attend such requests with a given maximum capacity. Each request should be transported by only one of these vehicles, that is, there is no transhipment. (Cordeau et al., 2008) define the same version of PDPTW as a Multi Vehicle one-to-one static Pickup and Delivery Problem with Time Windows. It is said to be multi-vehicle because allows more than one route, as opposed to single-vehicle variations where only one route is allowed. It is called one-to-one, because for each pickup request there is only one corresponding delivery. And it is a static version because all requests are known beforehand, while in the dynamic requests become available during the optimization process (Desrosiers et al., 1986; Psaraftis, 1988).

The PDPTW has a set of n transportation requests and all of them should be at tended. Each request has: (i) pickup location; (ii) delivery location; (iii) time window for pickup, indicating the earliest and latest time the pickup may be performed; (iv) time window for delivery; (v) service time, or how much time a vehicle takes to complete the service; and (vi) demand, how many units of goods the vehicle should pickup and deliver. The demand of the delivery location should be strictly complementary to the one of the pickup locations, i.e., if the pickup location has demand a, the delivery should have demand −a. It is important to distinguish between the types of time windows restrictions. These can be soft time windows, or hard time windows. The former considers a scenario where time windows can be violated, in order to perform all the deliveries. The latter considers the opposite scenario, where time windows cannot be violated, and a violation leads to an infeasible solution. The problem being studied considers only hard time windows. The fleet of m vehicles is located at a common starting location, referred as depot, from where vehicles start and end their routes. The problem considers that there is only one depot, and it has its own time window, defining the size of the planning horizon, or the maximum time a vehicle route can have. A solution to the PDPTW can be given in a graph as a set of vehicle routes, and a vehicle route is a set of ordered locations, or nodes, to be visited. The aim of the PDPTW is to find a feasible solution, so that the number of vehicles ($|s|$) is minimized, and the total cost of the routes is also minimized. This defines and hierarchical order of minimization, the first being the number of routes, and the second the total operational cost of the routes.

RESULTS AND DISCUSSION

This section presents a formal description of the PDPTW through a mathematical model in Mixed Integer Linear Programming (MILP) form, based on the work of (Grandinetti et al., 2014). The formulation considers the same objective function of (Lima et al., 2003), minimizing first the number of vehicles, and second the cost of all routes. As in other variations of VRP, the problem is defined on a graph $G = (V, A)$, where V is the set of all nodes and A the set of all arcs connecting two nodes. A usual

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PDPTW scenario has n requests and a maximum of m vehicles to be used. This defines the following sets: $P \subset V$ the set of all pickup locations, $D \subset V$ the set of all delivery locations and K the set of all vehicles, such that $|P| = |D| = n$ and $|K| = m$. Each node $p \in P$ has strictly one corresponding delivery pair, denoted as *dev(p)*; analogously, each node $d \in D$ has a unique pickup location. Two depots are considered in this model: the departure $\delta 0 = 0$, and the arrival $\delta 1 = 2n + 1$, which could be physically the same. Then, the set of all nodes is given by $V = P \cup D \cup \{\delta 0, \delta 1\}$. Also, each arc $a \in A$ connecting two nodes $i, j \in V$ has a time tij, and a cost
cij of using this arc. It is assumed the times and costs are non-negative, and that arc times satisfy the triangular inequality. Each node $i \in V$ has a service time si and a time window $[ei, li]$.

A vehicle is allowed to arrive at a location before service can start (before ei), but in this case it must wait until the start of time window to perform the visit. Though, a vehicle is never allowed to arrive after the maximum time li. Additionally, every node has a demand Qi associated, being $Qi > 0$ when $i \in P$, Qi i when $i \in D$ and $Qi = 0$ when $i \in$
$\{\delta 0, \delta 1\}$. This demand corresponds to the amount of goods a vehicle must pickup or deliver at the given location. The homogeneous fleet of vehicles has a maximum capacity U per vehicle. There is also a cost associated for allocating a vehicle to a route, given as a weight parameter ω. This weight should be big enough in order to dominate the objective function's value and drive the search to solutions with fewer vehicles. In practice, ω is defined according to the number of locations in the problem, so that it is able to dominate the value. Four sets of decision variables are used in this model: $xijk, i, j \in V, k \in K$, a binary variable which assumes one if the *arc (i, j)* is traversed by vehicle k, and zero otherwise; $yk, k \in K$, a binary variable which takes value one if vehicle k is used, and zero otherwise; $hik, i \in V, k \in K$, a real variable, which indicates the time vehicle k starts service at node i; and qik, $i \in V$, $k \in K$, a real variable indicating the remaining capacity of vehicle k before leaving node i. Both variables hik and qik are only well defined when vehicle k is used. Then, the mathematical model is given as follows:

Minimize $\omega \sum_{k \in K} y_k + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x_{ijk}$, (2.1)

Subject to

$\sum_{k \in K} \sum_{j \in V | j \neq i} x_{ijk} = 1$ $\quad \forall_i \in P$ (2.2)

$\sum_{j \in V | j \neq i} x_{ijk} - \sum_{j \in V | j \neq i} x_{dev(i)jk} = 0$ $\quad \forall_i \in P, k \in K$ (2.3)

$\sum_{j \in V | j \neq \delta 0} x_{\delta 0 jk} = y_k$ $\quad \forall k \in K$ (2.4)

$\sum_{j \in V | j \neq \delta 1} x_{\delta 1 jk} = y_k$ $\quad \forall k \in K$ (2.5)

$\sum_{i \in V} \sum_{j \in V | j \neq i} x_{ijk} \leq W_{yk}$ $\quad \forall_k \in K$ (2.6)

$\sum_{j \in V | j \neq i} x_{ijk} - \sum_{j \in V | j \neq i} x_{ijk} = 0$ $\quad \forall_i \in P \cup D, k \in K$ (2.7)

$h_{jk} \geq h_{ik} + (tij + si)xijk - W(1 - xijk)$ $\quad \forall_i \in j \in V | i \neq j, k \in K$ (2.8)

$h_{dev(i)k} \geq h_{ik} + t_{i\, dev(i)} \sum_{j \in V | j \neq i} x_{ijk}$ $\quad \forall_i \in P, k \in K$ (2.9)

$h_{ik} \geq e_i$ $\quad \forall_i \in V, k \in K$ (2.10)

$h_{ik} \geq l_i$ $\quad \forall_i \in V, k \in K$ (2.11)

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$$q_{jk} \geq q_{ik} + Q_j - W(1 - x_{ijk}) \quad \forall_i \in j \in V | i \neq j, k \in K \tag{2.12}$$

$$\max(0, Q_i) \sum_{j \in V | j \neq i} x_{ijk} < q_{ik} \quad \forall_i \in V, k \in K \tag{2.13}$$

$$\min(U, U + Q_i) \sum_{j \in V | j \neq i} x_{ijk} < q_{ik} \quad \forall_i \in V, k \in K \tag{2.14}$$

$$x_{ijk}, y_k \in \{0,1\} \quad \forall_i, j \in V, k \in K \tag{2.15}$$

$$h_{ik}, y_{ik} \in \mathbb{R} \quad \forall_i \in V, k \in K \tag{2.16}$$

As stated previously, the objective function (2.1) minimizes the accumulated costs. Parameter ω is responsible forrelating the cost of each route in solution to the number of vehicles used, resulting in the final total operational cost. Constraint (2.2) ensures that all requests are attended, while (2.3) ensures the pairing condition, that is, if a vehicle *k* serves a pickup *p*, then *k* must also serve the delivery pair of *p*, *dev(p)*. Constraints (2.4) and (2.5) assure that for eachroute, only one arc leaves depot $\delta 0$ and only one arc arrives at depot $\delta 1$, respectively. Constraints (2.6) link the *x*- variables to the *y*-variables: if an arc *(i, j)* is traversed by vehicle *k*, then *k* must be considered used (yk = 1); on the other hand, if a vehicle k does not traverse arc *(i, j)*, the corresponding *x* must be zero. Here, *W* is defined as a large non-negative scalar. Next, constraint (2.7) is the flow conservation constraint, stating that the number of incoming arcs must be equal to the number of outgoing arcs for all nodes, except the depots. Constraint (2.8) assures all nodes in a route are served after their predecessor, and constraint (2.9) ensures the precedence constraint, that is, a delivery node can only be served after its corresponding pickup node. Constraints (2.10) and (2.11) impose the time window limits for service to occur in node *i*, being the earliest time and latest time, respectively. Constraint (2.12) updates the remaining capacity of a vehicle before leaving node *i*. Constraints (2.13) and (2.14) assure that a vehicle's transportation will neither become negative nor exceed its maximum capacity *U*. Finally, constraint (2.15) ensures the binary condition of the *x* and *y*-variables, as well as, (2.16) set variables *h* and *q* to be real.

CONCLUSION

In this study, the authors propose a mathematical model in the form of Mixed Integer Linear Programming (MILP) from Pickup and Delivery Problems with Time Windows, in which goods must be transported from one location to another, taking into account time limits and certain vehicle capacities. This aims to minimize the number of vehiclesused, as well as operational costs for all routes.

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