

The search for alternative algorithms of the iteration method on a system of linear equation

Aam Jon Mintase Tarigan¹, Mardiningsih² Saib Suwilo³

Universitas Sumatera Utara, Medan, Indonesia

[1aamjonmintase221@yahoo.com](mailto:aamjonmintase221@yahoo.com), [2mardiningsih@yahoo.com](mailto:mardiningsih@yahoo.com), [3saibsuwilo@yahoo.com](mailto:saibsuwilo@yahoo.com)

Submitted : Apr 3, 2022 | Accepted : Jun 17, 2022 | Published : Oct 2, 2022

Abstract. The system of linear equations is a set of linear equations consisting of coefficients and variables. The coefficients in the system of linear equations exist in the form of real numbers and some are complex numbers. The system of linear equations has some form of solving or solution, ie a single solution, many solutions and no solutions. One of the most common problems encountered in systems of linear equations. Using modern mathematical methods, often a complex problem can be reduced to a system of linear equations. There are basically two groups of methods that can be used to solve a linear equation. The first method is known as the direct method, ie the method that searches for the completion of a linear equation in finite step. These are guaranteed to work and are recommended for general use. The second group is known as the indirect method or the method of iteration, which starts from an early settlement. Then try to fix almost in infinity, but convergent steps. The iterative methods are used to solve large Linear Equations Systems. And the proportion of zero is large, as are many systems encountered in the Linear Equation System. Therefore it takes an Alternative Algorithm in Iteration Method

Keywords: Alternative Iteration , Algorithm , Matriks .

INTRODUCTION

The system of linear equations is a set of linear equations consisting of coefficients and variables. The coefficients in the system of linear equations exist in the form of real numbers and some are complex numbers. The system of linear equations has some form of solving or solution, ie a single solution, many solutions and no solutions. One of the problems often encountered in Linear Equation Systems is the problem of finding solutions of a system of linear equations. (Aryani, 2012)

According to Irdam (2009), solving linear equations is one of the most important problems in mathematics, since more than 75 percent of all mathematical problems encountered in scientific and industrial applications involve the completion of a linear system to some extent. Using modern mathematical methods, often a complex problem can be reduced to a system of linear equations. In the real world, linear systems can be used to solve problems in some fields, including in the fields of commerce, economics, electronics, physics, chemistry and so forth.

In line with Marcel (2008) argue that the Solution of a system of linear equations is an important issue because linear equations are used in various fields of engineering and computer science. For example, linear equations are used in digital signal processing in the field of electrical engineering. Many engineering problems can be modeled into a linear system of equations. One example of problem solving with a system of linear equations is the powerful modeling and direction of electric current in electric circuits with Kirchoff law and mathematical modeling for the economics of a country or region based on various sectors of the economy using the Leontief model.

There are basically two groups of methods that can be used to solve a linear equation. The first method is known as the direct method, ie the method that searches for the completion of a linear

*name of corresponding author



equation in the finite step. Methods - This method is guaranteed to work and is recommended for general use. The second group is known as an indirect method or an iteration method, which stems from an almost initial settlement. Then try to fix almost in infinity, but convergent steps. Iteration methods are used to solve large Linear Equations Systems. And the proportion of zero is large, as are many systems encountered in the Linear Equation System. Therefore it takes an Alternative Algorithm in Iteration Method.

LITERATURE REVIEW

A Linear Persistence System consisting of n equations with n variables x_1, x_2, \dots, x_n , can be written in the form,

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}$$

System The above equation can be expressed by Matrix multiplication form, the system of linear equations can be solved by direct method or iteration method both methods have weaknesses and advantages, the chosen method will determine the accuracy of the system resolved. In the particular case of large-sized systems of linear equations, iteration methods are more suitable. Iteration method using Algorithm recursively, Algorithm is done to get the convergent value with tolerance given.

Based on the system of linear equations above can be in form

$$AX = b \tag{4.1}$$

Where , $A = [a_{ij}]$,

$$X = [x_j],$$

$$b = [b_j],$$

$$i = 1, 2, \dots, n$$

$$j = 1, 2, \dots, n.$$

Describe the Linear Equation System in such a way that is useful for developing iterative methods. Then obtained,

$$QX + (\hbar HA - Q)X = \hbar Hb, \tag{4.2}$$

Where \hbar = Additional Parameters ($\neq 0$)
Q = Splitting Matrix
H = Additional Matrices

Let W be the initial Estimate then equation (4.2) can be written as,

$$QX = W_0 + [(Q - \hbar HA)X + \hbar Hb - W_0] \tag{4.3}$$

Where,

$$L(X) = QX \tag{4.5}$$

$$C = W_0 \tag{4.6}$$

$$M(X) = [(Q - \hbar HA)X + \hbar Hb - W_0] \tag{4.7}$$

Using the New Digesting Technique, it draws on Daftardar-Gejji (2006) to form an Iteration method, where X represents,

*name of corresponding author



$$X = \sum_{i=0}^{\infty} X_i \tag{4.8}$$

Where M is described as,

$$M(X) = M(X_0) + \sum_{i=1}^{\infty} \left\{ M\left(\sum_{j=0}^i X_j\right) - M\left(\sum_{j=0}^{i-1} X_j\right) \right\}$$

Combining Equations (4.4), (4.8) and (4.9) we get the following equation,

$$L\left(\sum_{i=0}^{\infty} X_i\right) = C + M(X_0) + \sum_{i=1}^{\infty} \left\{ M\left(\sum_{j=0}^i X_j\right) - M\left(\sum_{j=0}^{i-1} X_j\right) \right\}$$

Using Equation (4.5) with (4.10) we find the following equation,

$$Q\left(\sum_{i=0}^{\infty} X_i\right) = C + M(X_0) + \sum_{j=1}^{\infty} \left\{ M\left(\sum_{j=0}^i X_j\right) - M\left(\sum_{j=0}^{i-1} X_j\right) \right\}$$

Thus, Following the Iteration Scheme as follows

$$Q(X_0) = C,$$

$$Q(X_1) = M(X_0),$$

$$Q(X_2) = M(X_0 + X_1) - M(X_0)$$

$$Q(X_3) = M(X_2 + X_1 + X_0) - M(X_1 + X_0)$$

Generally can be written in equation as follows,

$$Q(X_{m+1}) = M\left(\sum_{j=0}^m X_j\right) - M\left(\sum_{j=0}^{m-1} X_j\right), m = 1, 2, \dots \tag{4.12}$$

From Equation (4.7) with Equation (4.12) then we get,

$$Q(X_0) = C$$

$$Q(X_1) = (Q - \hbar HA) X_0 + \hbar Hb - W_0$$

$$Q(X_2) = (Q - \hbar HA) X_1,$$

$$Q(X_3) = (Q - \hbar HA) X_2$$

$$Q(X_{m+1}) = (Q - \hbar HA) X_m, m = 1, 2, \dots \tag{4.13}$$

From Eq. (4.13) is obtained.

$$\begin{cases} X_0 = Q^{-1}W_0 \\ X_1 = (I - \hbar Q^{-1}HA) X_0 + Q^{-1}(\hbar Hb - W_0) \\ X_m = (I - \hbar Q^{-1}HA) X_{m-1}, m=2,3,4,\dots \end{cases} \tag{4.14}$$

After drawing initial estimates $W_0 = \hbar Hb$ Then obtained,

$$\begin{cases} X_0 = \hbar(Q^{-1}H)b \\ X_m = (I - \hbar Q^{-1}HA)^m \hbar(Q^{-1}H)b \end{cases} \tag{4.15}$$

$m = 1, 2, 3, \dots$

Using Equation (4.8) with (4.15) we find the following equation

$$X = \sum_{k=0}^{\infty} X_k = \sum_{k=0}^{\infty} (I - \hbar Q^{-1}HA)^k \hbar(Q^{-1}H)b \tag{4.16}$$

Using Equation (4.16) we get the Iteration Scheme as the First Algorithm,

*name of corresponding author



$$X^{(k)} = \sum_{m=0}^k (I - \hbar Q^{-1}HA)^m \hbar(Q^{-1}H)b, k = 1, 2, 3, \dots$$

If $Q = D$

$$D = \begin{bmatrix} d_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & a_{n,n} \end{bmatrix}$$

$$X^{(k)} = \sum_{m=1}^k (I - \hbar D^{-1}HA)^m \hbar(D^{-1}H)b, k = 1, 2, 3, \dots$$

If $Q = D - \hbar L$ where,

$$D = \begin{bmatrix} d_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & a_{n,n} \end{bmatrix} \text{ and } L = \begin{bmatrix} 0 & 0 & \dots & 0 \\ -a_{21} & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ -a_{n1} & \dots & -a_{n,n-1} & 0 \end{bmatrix}$$

$$X^{(k)} = \sum_{m=1}^k (I - \hbar(D - \hbar L)^{-1}HA)^m \hbar(D - \hbar L)^{-1}Hb$$

$k = 1, 2, 3, \dots$

CONCLUSION

Using the new Decryption Techniques in finding Alternative Algorithms in Iteration methods to solve Linear Equation Systems is simpler than using the Homotopy Perturbation Method this method does not use Differential and is easy to implement.

REFERENCE:

- [1]. Bagnara, R.A. Unified Proof For the Convergence of Jacobi and Gauss-Seidel Methods. *Society for Industrial and Applied Mathematics*. Vol. 37,
- [2]. Daftardar, G. and Javari, H. (2005). Adomian decomposition: a tool for solving a system of fractional differential equations, *J. Math. Anal. Appl.* 301 508–518.
- [3]. Evans, D. J., Martins, M. M. dan Trigo, M. E. (2001). The AOR Iterative Method for New Preconditioned Linear System. *Journal of Computational Applied Math.* 132, 461-466.
- [4]. Jayachamarajandra, S. (2011). Eigenvalues of tridiagonal matrix using Sturm Sequence and Gerschgorin theorem, *Internasional Journal on Computer Science and Engineering (IJCSSE)*, India.

*name of corresponding author

