

Pareto Frontier Approach to Determining the Optimal Path on Multi-Objectives

Annisa Pratiwi^{1)*}, M.K.M. Nasution²⁾, E. Herawati²⁾

¹⁾Graduate Student of Mathematics Department, Universitas Sumatera Utara, Indonesia

^{2,3)}Mathematics Department, Universitas Sumatera Utara, Indonesia

¹⁾ annisapратиwi761@gmail.com, ²⁾ mahyuddin@usu.ac.id, herawaty.elv@gmail.com

Submitted : May 27, 2023 | **Accepted** : May 31, 2023 | **Published** : Jun 1, 2023

Abstract: Every issue we face in daily life can be resolved through mathematical modeling. The use of mathematical modeling to generate solutions frequently produces value that serves a single purpose. Sometimes a single-purpose function's solution does not offer the best solution value. In this study, the author models the multiobjective time-dependent vehicle routing problem using the Ant Colony Optimization (ACO) metaheuristic algorithm. The author then applies the pareto optimization principle to the determination of the optimal starting point for the route. An optimal Pareto frontier principle solution on a multi-objective model under control of the Ant Colony Optimization algorithm is the outcome of this study.

Keywords: Ant Colony Optimization; Multiobjective; Pareto Frontier; Time Windows; Vehicle Routing Problem

INTRODUCTION

Every issue we face in daily life can be resolved through mathematical modeling. The use of mathematical modeling to generate answers frequently produces value that serves a specific objective. Sometimes a single-purpose function's solution does not offer the best solution value. When there are two objective functions to be solved and taken into consideration so that it can offer an optimal solution, a mathematical model that focuses on a single objective function is unable to provide a solution. Due to the intricacy of the challenges, multiobjective applications—or, in other words, mathematical modeling with several purposes—have already started to replace single-objective applications. The target is the maximum-minimum that is computed combined. The problems that are resolved in multi-objective modeling can be linear or non-linear. The multi-variable goal function is converted into a single-purpose function model if the issue is a nonlinear multi-objective problem in order to identify the multi-objective model's balance point. In this study, the authors will use the Ant Colony Optimization (ACO) approach in the waste transportation process to apply the multi-objective question of control to the heuristic question of Vehicle Routing Problem (VRP). The process of moving waste from the temporary disposal node (TPS) to the final disposal site (TPA) is referred to as the waste transportation process. The issue with the trash transportation process is that choosing the route requires consideration of the carrying capacity, load weight, and trip distance. With the constraints that time and weight become barriers to be taken into account by the user, the VRP question will generate many solutions in the process of determining the best course. For these constraints, the ACO approach will be used in the formation of alternative solution mutations.

As time goes by to determine the optimal value of the resulting multi-objective solution, it is necessary to conduct a feasibility study rather than a resulting solution, this is due to the fact that based on the solution that has been produced, the distance between the objective functions must be minimized. The entire optimal solution produced is in the Pareto Frontier space. The pareto area is the ideal solution space for the complete objective function, but it occasionally deviates excessively from the ideal pareto

*name of corresponding author



zone tolerance. Pareto Optimality focuses on removing objective functions that do not yield optimal values in order to minimize the distances between the resulting solutions(Liu et al., 2019).

The association between the problem of automobile transportation with Temporary Disposal Places (TPS) has become a contemporary issue in various nations, notably in major cities. The research TPS VRP currently in place needs to be adjusted for a number of factors, including vehicle capacity (capacity), working hours (time window), and the presence of final disposal (TPA) as a necessary intermediate facility before the vehicle can return to the depot. Demand for various TPS exceeds vehicle capacity, allowing for multiple visits to one TPS with or without the use of the same vehicle (split service). While TPA handled the load, TPS handled the process of loading containers into the compactor truck. Split Service, Time Window, and Intermediate Facility Vehicle Routing Problem (VRPSSWIF)(Kumar, 2022).

By identifying the approach of the optimum solution set of the objective function and modeling to obtain the optimal resolution of the object function that is in the set of optimum solutions using the pareto frontier principle, this research will offer suggestions for how to approach the optimal solution of the multi-objective control panel. Simulation for the multifunct will be used to validate the model and analyze the resulting solution approach(Djunaidy et al., 2019).

LITERATURE REVIEW

Vehicle Routing Problem (VRP)

The Traveling Salesman Problem (TSP) is one of the most well-known and straightforward routing issues. TSP is a problem where a seller must travel to several cities and then return to the original city at the beginning. Routes are designed to reduce the amount of ground to be covered. The Vehicle Routing Problem (VRP) is an m-TSP with the presence of demand in each city and vehicle capacity. The difference is in the vehicle's capacity; if the TSP capacity is always assumed to be sufficient to meet the VRP, then certain solutions, such as multitrip systems, heterogeneous vehicles, etc., cannot be used define VRP as the process whereby a vehicle route is determined to satisfy the transportation demand with the fleet of vehicles offered at the lowest possible cost. In order to correctly execute all vehicle routes, the VRP is entrusted with deciding which vehicles handle which requests and how they are organized. Many VRP models contain special traits for certain sorts of models so they can be solved successfully.

A literature review is a critical, analytical summary and synthesis of the current knowledge of a topic. It should compare and relate different theories/research, findings, and so on, rather than just summarize them individually. It should also have a particular focus or theme to organize the review. In this section, the researcher can describe some of the related previous studies. Researchers can review the gaps in the research, then it can be used as a basis for research to be carried out(Camisa et al., 2021; Zhu et al., 2022).

Vehicle Routing Problem with Time Window

When it is considered that the time frame is constant, it displays opening and closing times that are greater than or equal to the service time for each customer. The service (loading and unloading) must wait if the truck arrives at a particular customer before the expiration time. Otherwise, the service can begin. The vehicle must arrive at the customer's location before or at the same time as the closing hour. Figure 1. depicts an example of a vehicle arriving at a client before it opens (Suprayogi & Priyandari, 2017). Each customer at VRPTW service must be attended to within a period of time known as a time window. Both a hard and soft time window might exist. A harsh time window occurs when a customer arrives too early and must wait until the service hours begin. Any infraction of the law will result in punishment(Ferliani et al., 2022).

*name of corresponding author



This is an Creative Commons License This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License.

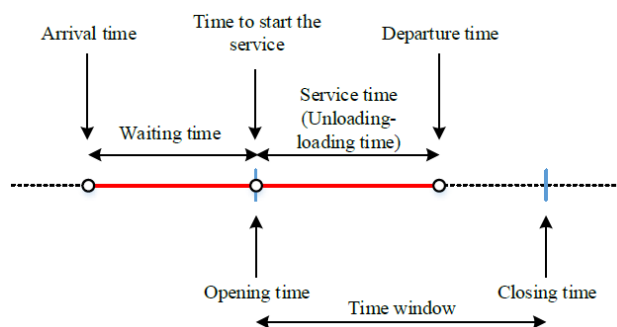


Figure 1. Time Wondows Process

Pareto Frontier

Pareto Frontier is one of the ways to identifying a set of objective solutions in an optimal objective functional space. The optimal solution approach is achieved by determining the optimal solution set by finding objective functions that dominate the objective function in a suitable solution space, the determination of the optimal object functions is obtained when there are objective funtions that are on the convex pareto frontier line. Multi-objective optimization employing the Pareto Frontier principle can be framed as a question of decision-making from optimization values moving simultaneously from two or more objective functions(Ferliani et al., 2022; L’Hostis, 2017; Lin et al., 2019).

Definition 1. The vector $F(x)$ is considered to be predominant of the $F(y)$ vector if and only if $f_i(x) \leq f_i(y)$ for each $i \in \{1,2, \dots, n\}$ and $f_i(x) < f_y(x)$ for each . The point $x \in C$ is said to be in the optimal pareto local if and only if there is an open equation $B(x)$ of x such that $F(x)$ for all $x \in B(x) \cap C$. If $F(x^*)$ dominates over $F(x)$ for all $x \in C$, then x^* is said to be the global pareto optimal.

Based on Definition 1, it is determined that the lens that is in the objective space in the convex curve is able to produce a viability solution that dominates its initial solution. Therefore, local and global determination of the region is vital to enable decision-making. Additionally, it is considered that $F: \mathbb{R}^n \rightarrow \mathbb{R}^N, h: \mathbb{R}^n \rightarrow \mathbb{R}^{N_e}$ and $g: \mathbb{R}^n \rightarrow \mathbb{R}^{N_i}$ are variables that can be deducted twice. Where n is assumed to be the sum of the variable, N is the total of the objective, N_e and N_i respectively are the amount of the equation and the barrier equation. A respectable solution is a solution of the objective function that is within the range of the Pareto frontier area.

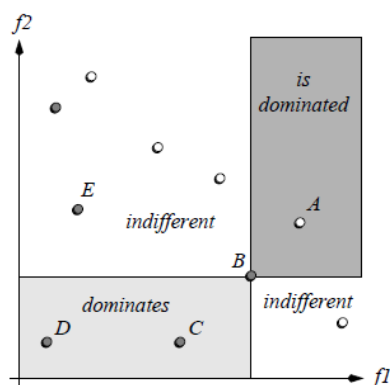


Figure 2. Pareto Domination Space

In identifying the solution of the multi-objective approach apply the approaches weighted sum method, ϵ -constraint, and weighted metric space. The purpose of each method is to determine the approach to addressing a multi-objective problem.

*name of corresponding author



METHOD

At the development stage of the model there are 3 important steps in it, namely: the development of the mathematical model as a model that represents the conditions of the real system and is used as a reference for the creation of models with the approach of the ACO algorithm, development of a model with the algorithm used in the search for optimal value on this research and finally the model will be verified and validated in order to be able to solve the problem on this study. Before generating mathematical models and ACO algorithm models, conceptual models will be established as the basis for making other models and to inform the true conditions of the system so that it is easier to grasp. There are a number of constraints that must be completed into the established model, among them are global restrictions that need be met.

Numerical Example

Waste transportation begins from the truck departing from the location of the depot, which is the main point and continues with the loading process on the TPS. The compactor truck has an assumed capacity of 35 containers, when the capacity is already full the vehicle will carry out the unloading procedure on the TPA. Loading is constantly expected to take 1 minute per container and emptying is assumed constantly to take 60 minutes. The time of departure of the vehicle is adjusted to the first TPS to be visited by the vehicle. The vehicle can carry out the process of transportation back to the next TPS after carrying out the unloading process with the condition that the vehicle must return to the depot before or right at 15.00 WIB. The challenge with waste transport routes is the number of vehicles that are not optimal to fulfill the demand for transportation in the entire existing TPS, especially when at the time there are damaged cars. On the other hand, the demand for garbage transportation in each TPS is increasing in accordance with the increasing population expansion, therefore there is a need for optimization as well as tools utilized as considerations in selecting the route well and fast. Each TPS location has a varied number of requests that must be served by vehicles visiting the site with the limits of vehicle capacity and also depot service time. Compactor trucks have an assumed capacity of 35 waste bins on each TPS. All cars must return at 3 p.m. to the Depot. The number of requests in some TPS is larger than the capacity of the vehicle, so for each TPS can be visited by more than once. This data is then utilized to see the spread of requests that must be served every day by a compactor truck.

Table. 1 Request of Costumer

NO	Request of Costumer	NO	Request of Costumer	NO	Request of Costumer	NO	Request of Costumer
1	140	21	12	41	2	61	20
2	13	22	55	42	2	62	22
3	10	23	4	43	3	63	35
4	2	24	6	44	2	64	140
5	1	25	90	45	15	65	70
6	2	26	40	46	24	66	70
7	1	27	20	47	23	67	70
8	140	28	15	48	33	68	80
9	70	29	40	49	40	69	62
10	70	30	7	50	25	70	70
11	60	31	4	51	110	71	70
12	20	32	30	52	140	72	70
13	108	33	50	53	35	73	80

*name of corresponding author



NO	Request of Costumer	NO	Request of Costumer	NO	Request of Costumer	NO	Request of Costumer
14	23	34	12	54	15	74	6
15	15	35	1	55	35	75	6
16	1	36	2	56	20	76	15
17	70	37	5	57	20	77	1
18	30	38	10	58	20	78	30
19	20	39	4	59	20	79	18
20	20	40	2	60	20	80	20

RESULT

Capacity Constraint

Capacity constraints need to be developed because capacity becomes an important factor in terms of meeting demand, this development needs to be done because of the differences in indices and variables that must be adjusted so that the case of garbage transportation with the VRP model can be modeled and adapted. In this section, the researcher will explain the results of the research obtained. Researchers can also use images, tables, and curves to explain the results of the study. These results should present the raw data or the results after applying the techniques outlined in the methods section. The results are simply resulting; they do not conclude.

$$\sum_{i \in 0} d_{il} = 0, \forall l \in k \quad (1)$$

The capacity of all vehicles coming out and entering the depot has 0 loads then,

$$Q_i Y_{il} > q_{il}, \forall i \in V_c \quad (2)$$

$$\sum_{i \in V_c} x_{ijl} = y_{il}, \forall i \in V_c, l \in k$$

Both constraints guarantee that the binary variable will be a positive value when the material is taken on the node i using the vehicle l ,

$$d_{il} + q_{il} \leq d_{jl} + (1 - x_{ijl})M, \forall i, j \in V_c, j \in l \in V, l \in k \quad (3)$$

Then accumulate the request for each customer node,

$$d_{il} \leq C, \forall i \in V_c, l \in k \quad (4)$$

The impediment in the process of fulfilling the request must not exceed the capacity of the vehicle,

$$\sum_{l \in k} q_{il} = Q_i, \forall i \in V_c, l \in k \quad (5)$$

The sum of requests met by the vehicle $l \in k$ on the customer node is equal to the value of the request held by the node $\forall i \in V_c$,

$$\sum_{i \in m} d_{il} = 0, \forall i \in V_c \quad (6)$$

The capacity of all vehicles out of the TPA has 0 loads,

$$\sum_{i \in V} Q_i = Q_m, \forall i \in V_c \quad (7)$$

The total demand on the supply node is equal to the total transport capacity on the TPA,

$$\sum_{i \in g, g'} d_{il} = 0, \forall l \in k \quad (8)$$

Capacity of all vehicles coming out and entering the fuel filling point that has a capacity of 0 loads.

See also:

Q_i : Vehicle capacity on the node- i

q_{il} : Request capacity of node- i on vehicle l

*name of corresponding author



d_{il} : Distance of node-i on vehicle l

V_c : Vehicle transport capacity

x_{ijl}

Route of vehicle node-i to vehicle l

Y_{il} : Total transport capacity of the node-i vehicle

y_{il} : power on the node-i of the vehicle

C : Total demand capacity

ACO algorithm for VRPSSTWIF

The application of Ant Colony Optimization on the shortest route search will provide a solution to the route passed by the vehicle in transporting cargo according to customer requirements, the obstacles that occur will result in the need for the development of algorithms in determining the route with the Vehicle Routing Problem principle adopted from the ACO algorithm. The development of the algorithm is described in Algorithm 1 below.

ACO algorithm for VRPSSTWIF

Input: number of ants N, distance c_{ij} (other than the TPA node), maximum iteration

Output: Best TSP route, total TSP distance

Foreach a = 1: iter do

Reduction of evaporation where $e=e-(e-e_r)*a/iter$

Foreach i=1: n do

Starting the journey from node 1

End

Foreach i=1:n do

Calculate the visibility value

Foreach j=1:n do

The process of converting the selected node to zero, calculating the probability of the selected node, Creating a Random Number

Foreach k=1:n do

Calculating the cumulative chances.

Select a node that has not been visited $r \leq s, s = \text{cum } P(k)$

End

End

End

End the search for node travel routes

Foreach i=1:n do

Initial distance = 0

Foreach k=1:n do

Total distance = starting distance + distance from node 1 to j + distance of node i to j + distance from city j to i

End

Total distance for the route generated by each ants

End

Choose the smallest total distance from the entire journey of the ants.

The journey route is the route that produces the smallest distance the entire ants have traveled.

Foreach i=1:n do

Foreach j=1:n do

Additional number of chromosomes

Update of new chromosome values

End

End

Choose the smallest node distance from the entire journey

End

*name of corresponding author



Parameter Test

Once the algorithm model is confirmed and verified, then the algorithm is ready to be used for larger case samples. This experiment demands testing numerous times the sample with several replications for each candidate parameter to be picked. After the parameter is decided, a model will be run with that parameter in numerous replications to know the best value of the function purpose. The findings will be analyzed using the ACO algorithm. Within the ACO algorithm contains random values therefore in deciding the value of many parameters need to be done several times comparison of parameter values in several replications to identify which parameter value is better to use in search.

Table. 2 Parameter test

ρ	N	i	Min	Mean	SD	ρ	N	i	Min	Mean	SD		
0.5	5	500	4304	4422.8	75.11	0.6	5	500	4322.3	4406.4	46.91		
			35	35.7	0.67				35	35.9	0.57		
			2.7188	2.8141	0.07				2.75	2.8797	0.12		
		1000	4306.6	4369.4	30.45			1000	4353.6	4423.8	61.31		
				35	35.6					0.52	35	35.9	0.74
				5.1094	5.2953					0.13	5.375	5.586	0.16
			1500	4336.7	4404.4				59.38	1500	4325.8	4421.2	78.51
				35	35.9				0.57		35	36	0.47
				7.625	8.0682				0.44		8.4531	8.6	0.11
	10	500	4335.6	4402	47.44		10	500	4301.3	4365.3	48.22		
			35	35.8	0.63				35	35.3	0.67		
			5.25	5.425	0.12				5.7969	7.0172	2.97		
		1000	4283.4	4426	85.11			1000	4310.4	4413.5	50.74		
				35	35.5					0.53	35	35.8	0.42
				10.203	10.684					0.28	10.125	10.558	0.55
			1500	4259.2	4359.7				57.39	1500	4277.8	4384.8	53.86
				35	35.2				0.42		35	35.8	0.63
				15.125	15.534				0.33		15.203	15.948	0.74
	15	500	4315.1	4386.3	56.21		15	500	4375.4	4422	46.84		
			35	35.7	0.67				34	35.6	0.84		
			7.9531	9.3422	2.91				8.0313	8.425	0.30		
		1000	4302.4	4387	56.12			1000	4220	4337.1	77.31		
				35	35.4					0.52	34	34.6	0.52
				16.406	17.77					1.37	16.078	16.488	0.21
1500			4295.7	4367	42.74	1500			4290.5	4372.6	45.00		
			35	35.5	0.71				35	35.7	0.48		
			22.609	23.091	0.29				24.359	25.578	0.76		

*name of corresponding author



Route Recommendation Simulation VRPSSTWIF

By running a simulation with the circumstances of service from the VRP query, then a recommendation of the route can be taken by the vehicle is produced.

No	Route VRP	Time in (Minute)	Time out(Minute)	Distance
1	1-15-16-82-16-9-82-1	294	747	109.45
2	1-9-82-9-82-1	291	737	108.3
3	1-9-82-9-73-82-1	291	737	108.4
4	1-73-82-73-17-12-82-1	292	730	105.8
5	1-12-34-82-34-82-34-35-82-1	289	885	150.6
6	1-28-29-82-14-82-1	289	726	92.8
7	1-14-82-14-82-14-82-1	282	897	138.4
8	1-46-13-82-18-82-1	279	770	121.4
9	1-18-82-59-60-82-1	283	772	120.3
10	1-60-62-61-82-61-58-49-82-1	288	801	125.22
11	1-49-50 82-50-33-82-1	279	826	136.9
12	1-33-74-82-74-82-1	289	759	115.6
13	1-74-82-74-63-64-82-1	283	747	111.75
14	1-6-10-82-10-82-10-82-1	289	884	151.9
15	1-21-43-56-82-56-23-82-1	289	745	99.75
16	1-23-82-23-30-82-30-82-1	293	899	134.95
17	1-40-25-42-48-82-41-39-38-24-22-5-82-1	289	741	102.04
18	1-3-53-82-53-82-1	290	694	93.8
19	1-53-82-53-82-53-82-1	288	837	137.7
20	1-27-82-27-4-70-82-1	293	744	94.25
21	1-70-82-70-69-82-69-82-1	281	860	138.9
22	1-69-65-82-65-82-65-82-1	284	845	128.6
23	1-65-82-65-47-82-47-82-1	277	818	130.9
24	1-52-82-52-82-52-82-1	281	829	124.6
25	1-52-11-82-1182 11 54 82 1	281	861	137.7
26	1-54-51-32-31-82-31-8-6-7-19-82-19-79-82-1	283	888	157.45
27	1-75-20-81-82-81-77-55-82-1	271	777	115.57
28	1-55-76-72-82-72-82-72-78-82-1	274	896	161.75

*name of corresponding author



This is an Creative Commons License This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License.

29	1-80-26-82-26-82-1	270	730	108.3
30	1-26-82-26-71-82-1	283	720	102.8
31	1-71-82-71-68-82-68-82-1	282	880	155.9
32	1-68-37-45-2-82-2-82-1	286	724	112.5
33	1-2-82-2-82-2-44-82-1	294	898	155.1
34	1-57-66-82-66-82-66-82-1	274	847	135
35	1-67-82-67-82-36-82-1	274	886	183.2

Pareto Optimal Solution

The final vehicle route solution does not guarantee that the resulting solution is the optimal solution, so it is required to assess the determination of the optimal solution using the Pareto frontier principle. The Pareto Frontier concept would plot objective solutions derived both from the total distance and time created in a route to then perform the principle of the dominance of each objective function. The objective solution on the Pareto line is the ideal solution that can be chosen in the determination of the optimal route.

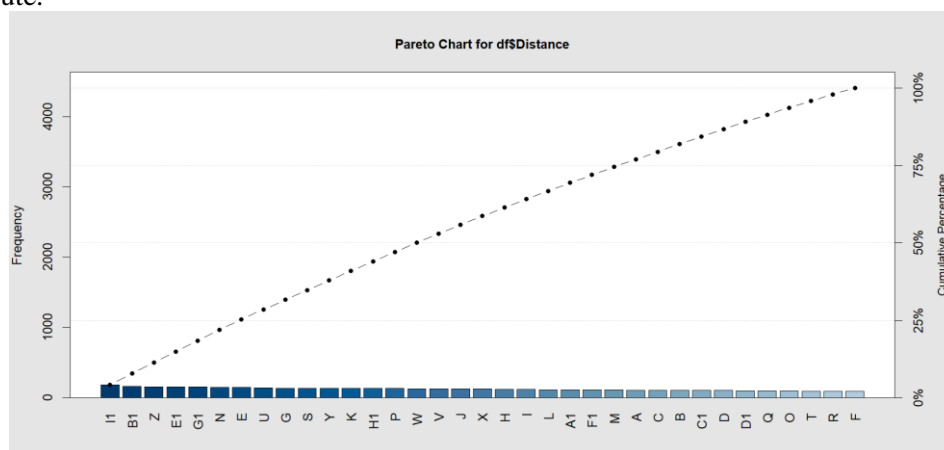


Figure 3. Pareto Chart Optimality

The pareto chart produced in Figure 4.4 shows that the route I1 is the route that has the largest cumulative percentage that can be interpreted as being the non-domination of the objective solution of the problem of vehicle travel route.

DISCUSSIONS

We have modeled multi-objective control problems with the development of vehicle routing problems with restrictions on time windows, split services, and intermediate facilities. This modeling is carried out by involving metaheuristic issues using an algorithm of colony optimization that aims to make barriers to the function of the target an initial solution in problem solving.

CONCLUSION

Based on the research carried out, it can be concluded that the modelling of constrained multiobjective optimization performed by applying the metaheuristic question of Ant Colony Optimization (ACO) then connected with the Pareto Frontier principle provides an optimal discreet solution through the principle of non-domination applied to the pareto front principle. Multi-objective problems do not provide distinct solutions and even produce multiple objective solutions so that by applying non-domination pareto front results in optimal travel routes according to the balance between transport capacity and service time.

*name of corresponding author



ACKNOWLEDGMENT

I thank you Prof. M.K.M. Nasution and Prof. Elvina Herawati as my supervisor that lead this research of Theses.

REFERENCES

- Camisa, A., Farina, F., Notarnicola, I., & Notarstefano, G. (2021). Distributed constraint-coupled optimization via primal decomposition over random time-varying graphs. *Automatica*, *131*, 109739. <https://doi.org/10.1016/j.automatica.2021.109739>
- Djunaidy, A., Angresti, N. D., & Mukhlason, A. (2019). Hyper-heuristik untuk Penyelesaian Masalah Optimasi Lintas Domain dengan Seleksi Heuristik berdasarkan Variable Neighborhood Search. *Khazanah Informatika: Jurnal Ilmu Komputer Dan Informatika*, *5*(1), 51–60. <https://doi.org/10.23917/khif.v5i1.7567>
- Ferliani, M., Schmidt, S., Schulz, V., Binois, M., Picheny, V., Vodopija, A., Tušar, T., Filipič, B., Roostapour, V., Neumann, A., Neumann, F., Friedrich, T., Personal, M., Archive, R., ESTECO, Roberts, M. C., Dizier, A. S. T., Vaughan, J., Šcap, D., ... Lu, X. (2022). Determination of the pareto frontier for multiobjective optimization problem. *Artificial Intelligence*, *2*(10), 103597. <https://doi.org/10.1016/j.artint.2021.103597>
- Kumar, S. (2022). Modeling usage intention for sustainable transport: Direct, mediation, and moderation effect. *Sustainable Production and Consumption*, *32*, 781–801. <https://doi.org/10.1016/j.spc.2022.05.019>
- L'Hostis, A. (2017). Detour and break optimising distance, a new perspective on transport and urbanism. *Environment and Planning B: Urban Analytics and City Science*, *44*(3), 441–463. <https://doi.org/10.1177/0265813516638849>
- Lin, X., Chen, H., Pei, C., Sun, F., Xiao, X., Sun, H., Zhang, Y., Ou, W., & Jiang, P. (2019). A pareto-efficient algorithm for multiple objective optimization in e-commerce recommendation. *RecSys 2019 - 13th ACM Conference on Recommender Systems*, 20–28. <https://doi.org/10.1145/3298689.3346998>
- Liu, Y., Ishibuchi, H., Nojima, Y., Masuyama, N., & Han, Y. (2019). Searching for Local Pareto Optimal Solutions: A Case Study on Polygon-Based Problems. *2019 IEEE Congress on Evolutionary Computation, CEC 2019 - Proceedings*, 61876075, 896–903. <https://doi.org/10.1109/CEC.2019.8790066>
- Shan, S., & Wang, G. G. (2005). An efficient Pareto set identification approach for multiobjective optimization on black-box functions. *Journal of Mechanical Design, Transactions of the ASME*, *127*(5), 866–874. <https://doi.org/10.1115/1.1904639>
- Zhu, S., Sun, H., & Guo, X. (2022). Cooperative scheduling optimization for ground-handling vehicles by considering flights' uncertainty. *Computers and Industrial Engineering*, *169*(March), 108092. <https://doi.org/10.1016/j.cie.2022.108092>

*name of corresponding author



This is an Creative Commons License This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License.