SinkrOn
JURNAL & PENELITIAN TEKNIK INFORMATIKA
Sinkron : Jurnal dan Penelitian Teknik Informatika
Volume 7, Number 3, Juli 2023
DOI : https://doi.org/10.33395/sinkron.v8i3.12489
e-ISSN : 2541-2019
p-ISSN : 2541-044X

A Mathematical Approach to Dampening Sea Waves Using Submerged Permeable Breakwater

J.L. Marpaung[1], Tulus[2)*], Parapat Gultom[2]
[1]Magister of Mathematics, Universitas Sumatera Utara, Indonesia
[2,3]Mathematics Department, Universitas Sumatera Utara, Indonesia
[1]jomarpaung4@email.com, [2]tulus@usu.ac.id, [3]parapat@usu.ac.id,

Submitted : May 31, 2023 | **Accepted** : Jun 6, 2023 | **Published** : Jul 1, 2023

Abstract: A wave is an energy that can propagate with a medium; the propagation of a wave moves with respect to time by carrying energy that moves with velocity per unit of time. Sea waves are one of the propagating wave problems that are broken down to produce wave propagation with a relatively inhomogeneous minimum amplitude and speed of sea waves, which have their own difficulties in solving them numerically. This study aims to analyze the stability of wave propagation on submerged breakwaters. This research approximates the finite discretization of the breakwater domain and then combine it with the Finite Element Method to determine the moving elements of the velocity of fluid flow through a porous submerged breakwater. The result of the research is the equation of the inflated wave and the simulated representation displayed on the wave breakdown process, the point that becomes the center of the waves breakdown will give a focused red color indicator meaning there is a change in momentum and potential energy that occurs and then changes the colour of the post-flattering of the sea wave so that the sinking wave breaker is a method to obtain the minimum speed and amplitude values that can be used for coastal engineering.

Keywords: Breakwater, Finite Element Method, Modelling, Navier-Stokes Equation, Simulation

INTRODUCTION

A wave is a propagating vibration that propagates in a medium or a set of objects that interact with each other. Real examples of waves can be seen in surface wave problems, such as ocean waves on the beach. Waves on the surface of the water are a process of interaction between moving air masses and the surface layer of water. This interaction produces a pattern of peaks and valleys that is influenced by the presence of energy and momentum. Basically, surface water waves are not only caused by air masses; these waves can also be caused by other activities that exist at the bottom of the water depth. (Bai et al., 2022) explain that the propagation speed of surface water waves depends on the water's surface tension, hydrostatic pressure, bottom depth, mass density, and gravity. The waves that occur on the surface of the water are caused by several things, such as wind blowing or vibrations at the bottom of the water, for example, tsunami waves. Kounadis (2020) studied the use of a porous beam as a wave breaker to lessen the force of waves on the surface of sea water. The initial step in the investigation was to create a porous beam model of the surface wave of water that travels through the wave breaker (Kounadis & Dougalis, 2020). The continuity equation, momentum equations, potential of the wave speed, fluids in porous media, Laplace and Bernoulli equations in fluids and porous medium, as well as the kinematic and dynamic finite conditions on the surface of the porous beam were used to form the model. The Limit Element Method is one of the methods that can be used to solve numerous kinds of differential equations. The

*Corresponding Author: Tulus

SinkrOn
JURNAL & PENELITIAN TEKNIK INFORMATIKA
Sinkron : Jurnal dan Penelitian Teknik Informatika
Volume 7, Number 3, Juli 2023
DOI : https://doi.org/10.33395/sinkron.v8i3.12489
e-ISSN : 2541-2019
p-ISSN : 2541-044X

resolution of the equation is made by discretizing the equation with the initial condition and the limit condition, subsequently determining the stability of the discretization of the equation. Tulus (2019) use the method of the finite element as a method based on the expansion of the Taylor row(Tulus, Khairani, et al., 2019). One way to use different methods is using an implicit scheme. The implicit approach has the virtue of being unconditionally stable. Step time Δt can be taken nevertheless (big) without creating instability.

The goal of the wave breakers research is to find the phenomena that arise in opposition to the force created by the movement of seawater toward a shallow coastal area. (Bao et al., 2022). Several mathematical physicists have developed research on wave breakers, concentrating on the wave breaker model. (Winarta et al., 2018) According to the established model parameters, wave splitters with dimensional discrepancies would yield various results in research that have been done. The wave breaker model is affected by a number of critical factors that affect optimization, including big wave speed, wave height, depth, pressure, and even Newton's law, which causes momentum to occur on the wavelength breaker. In his research, he detailed how the development of the Navier-Stokes equation and analysis of the interaction between models of the wave breaker used on a three-dimensional problem with the model of the porous wave broker led to a 3D wave retardation model for the surface-appearing wavebreaker. To determine how much wave reduction is produced from the value of the wave's speed and height, wave breakers are frequently simulated. (Khater & Botmart, 2022; Rupali & Kumar, 2021) Because it demands a multipurpose programming model, simulation involving various parameters will have a complex solution. (Han & Wang, 2021; Yu & Huang, 2022). The authors will use the finite element approach to do a computer analysis of stability on a wavelength for the problem of sinking wave breakers with porous models in this study. The authors propose that the sinking wave breaker will act as a momentum barrier for the current beneath the surface, reducing the large momentum that happens at the bottom of the shallow area before the wave crashes into the coastal area. The study's findings include a massive simulation of the wave height (h) of the depth fluctuation of its wave breaker position from the sea surface. In addition, the study will mimic the effect of speed change on the huge wave amplitude that develops. In order to construct a model of a sinking wave breaker, reliability testing will be undertaken that moves against time to a stability area against the modification of the barrier performed.

LITERATURE REVIEW

A literature review is a critical, analytical summary and synthesis of the current knowledge of a topic. It should compare and relate different theories/research, findings, and so on, rather than just summarize them individually. It should also have a particular focus or theme to organize the review. In this section, the researcher can describe some of the related previous studies. Researchers can review the gaps in the research, then it can be used as a basis for research to be carried out

Wave

The interaction between moving air masses and the water's surface layer results in waves on the surface of sea water. The presence of energy and momentum affects the patterns of peaks and valleys produced by these interactions. In general, sea surface waves can also be created by various processes occurring at the bottom of the depths in addition to air masses. In fact, there are many types of waves that may be identified when they occur in the water. Depending on the manner that causes it, this type or these types of waves vary.

Wave Breakwater

A wave breaker, also known as a breakwater, is a prasarana constructed to break ocean waves by absorbing some of the wave force. The beach is crushed by abrasion, which is controlled by the wave breaker. The wave breaker in this scenario is supposed to be a porous balloon. The following illustration shows how this wave breaker simulation looks. The porous media is positioned at the bottom of the water surface canal according to the simulation shown in the above image. The goal of this is to drastically cut the wavelength on the water's surface. The waves that travel through porous material are absorbed, which interrupts the wave flow and alters the wavelength. The disruption's amplitude

*Corresponding Author: Tulus

Sinkron : Jurnal dan Penelitian Teknik Informatika
Volume 7, Number 3, Juli 2023
DOI : https://doi.org/10.33395/sinkron.v8i3.12489
e-ISSN : 2541-2019
p-ISSN : 2541-044X

decreases, making the surface waves less dangerous. The optimum fluid flows beneath the porous layer that extends down the canal's base. Hydraulic conductivity, or k, the speed of the fluid particle in the porous media, is a property of the porous layer. The depth of the fluid layer is $y = h(x)$ from the wave's surface to the interface, and the depth of the channel is $y = d(x)$ from the surface to the canal's bottom. The potential functions, and ϕ' for fluids in the porous media are utilized for the fluid layer and the presumed irrotational fluid flow, respectively. Use the equation $\phi' = -k\ (yg\rho\ +\ p)$, where p, ρ and g stand for pressure, density, and gravitational acceleration, respectively. The porous material at the base of the channel has a thickness of $y = -d(x) + h(x)$, where $-d(x) > -h(x)$.

Navier-Stokes Fluids Modelling

A mathematical field called the Navier-Stokes equation formulates the fluids question in terms of both fluids and fluid properties. The Navier-Stokes equation model explains how the component u, v on the x, y coordinates that move against time intersects the speed vector values. The definition of vector analysis states that all fluid quantities are functions of space and time, and that fluid quantities can take the form of vector values for acceleration (v) and speed (v). (a).

$$\vec{a}_x = \frac{du}{dt} = \frac{\partial u}{\partial t} + \vec{v}.\nabla u \quad (1)$$
$$\vec{a}_y = \frac{dv}{dt} = \frac{\partial v}{\partial t} + \vec{v}.\nabla v$$

The expression for a can be expressed as follows if the value of v on equation is separated:

$$\vec{a} = \left(\frac{\partial}{\partial t} + \vec{v}.\nabla\right)\vec{v}$$

In order to provide the best wave retention simulations, the study employs a series of computational analysis techniques based on the wave retardation optimization process. To find the elemental and overall solutions of a model function, the general wave retardation equation against the permeable wave breaker will be researched in the literature using the finite element approach. The end result is a computational analysis that compares a wave model to a stable permeable wave breaker and displays a simulation of waves with a significant rapid drop of the wave's minimum. When conducting research on airborne pollution, several steps will be taken, including: examining the standard water wave equation; determining the baseline and baseline conditions of the model water wave equation; and, finally, modifying the water wave equation by adding a variable designed to produce a negative reading of the water wave equation (u_{tt}) nilai. The next step is to run a simulation using COMSOL Multiphysics 5.6 based on the elements up to the point where it is combined with the finite element against the domain of the wavelength breaker model in order to obtain a representation of the distribution of amplitude, speed, and energy with various wave freeze simulation iterations. This is done after the wave equation with the ocean wave breaker has been modified.

METHOD

The research employs a series of computational analysis techniques based on the wave retardation optimization process, which is critical in developing optimal wave retention simulations. Model discretion will be used to conduct a literature analysis of the general wave retardation equation against the permeable wave breaker using the boundary element approach to produce the element solution and the global solution of a model function. The end result is a computational examination of a wave model against a stable permeable wave breaker that demonstrates a simulation of waves with a big rapid drop of the arriving wave's minimum. In the conclusion of the research on shallow water, waves will be several steps of solution that are to be discretization of the wave equation of water in general then made the determination of the initial value and the limit conditions of the model of the wavelength equation, the next step is to modify the waves equation with the addition of the variable wave breaker that aims to give a negative value of the acceleration of seawater wave u_{tt} the negative value is assumed

*Corresponding Author: Tulus

Sinkron : Jurnal dan Penelitian Teknik Informatika
Volume 7, Number 3, Juli 2023 e-ISSN : 2541-2019
DOI : https://doi.org/10.33395/sinkron.v8i3.12489 p-ISSN : 2541-044X

as the excellent retention of sea wave should be opposed from the direction of the wind wave so that will reduce the value of wavelength (λ) and wave speed (v) as well as the momentum of the p_x wave (t). Once the wave equation with the ocean wave breaker has been modified, the next step is to perform a simulation using COMSOL Multiphysics 5.6 based on elements up to which it is combined with the finite element against the domain of the wavelength breaker model to obtain a representation of the distribution of amplitude, speed, and energy with several variations of wave freeze simulation(Tulus et al., 2020; Tulus, Marpaung, et al., 2019).
The stages in the solution carried out in this study are:

1. Get data that is wave breakers information acquired from books or specific papers on stability study on wave breaker.
2. Identify the problem of turbulent wave delay to a stable wave retardation to be studied, i.e. in critical conditions that result in large amplitude values for the wavelength gap.
3. The method is finite-boundary element method.
4. Analysis of the influence of submerged permeable breakwater in keeping the flow rate increased.
5. Solve the Navier-Stokes 2D equation of modified acceleration by the wave breaker.
6. The separation of a partial differential equation into an ordinary difference equation.
7. Conducting tests with computations that provide numerical values against mathematical models obtained from computer analysis of wave retardation.

RESULT

a belief that the ocean is large and homogeneous with a huge area of penetration. The stem will vibrate as a result of a style that depends on the time $P_x(t)$ that operates on it, creating an internal wave that swings back and forth. Assuming that a constant wind blows over the sea's surface is necessary for such a style to function at sea level. Observing a tiny section of the wave on the surface (i.e., from point x to $x + \Delta x$) on the x axis will help you identify styles that function on moving waves.

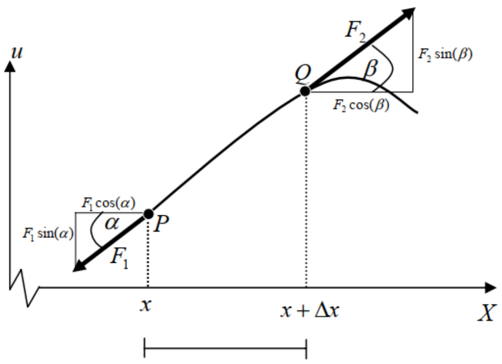

Figure 1. Projection of Wave Forming Style

Differential equations on ocean waves can be obtained by looking at the styles that work on the part of the sea surface (at the gap $(x, x + \Delta x)$). Tension on the ocean level is a line at each point on the sea level. Where F_1 is a style that works at the point P and F_2 is a styl that works on the point Q. The projection of the style that works on the x-axis is $F_1 \cos\alpha$ and $F_2 \cos\beta$. Note that each point on the surface is only moving vertically and nothing moves horizontally, so the style which works in the horizontal direction must be constant. With the resulting style rule on the x-axis style projection, then $F_1 \cos\alpha = F_2 \cos\beta = F$. Where $\rho\Delta x$ is the mass of the part of the sea water and u_{tt} is the second derivative of the position of the object that is the value of the acceleration of the body. it can be written that:

$$\tan\beta - \tan\alpha = \frac{\rho\Delta x}{F} . u_{tt} \qquad (2)$$

The values of tan β and tan α are gradients on x and $x + \Delta x$. Then it can be written that $\tan\alpha = u_x(x, t)$ and $\tan\beta = u_x(x, x + \Delta x)$. Can be written as:

*Corresponding Author: Tulus

Sinkron : Jurnal dan Penelitian Teknik Informatika
Volume 7, Number 3, Juli 2023 e-ISSN : 2541-2019
DOI : https://doi.org/10.33395/sinkron.v8i3.12489 p-ISSN : 2541-044X

$$tan\ \beta - tan\ \alpha = \frac{\rho \Delta x}{F}.u_{tt} \tag{3}$$

$$u_x(x, x+\Delta x, t) - u_x(x,t) = \frac{\rho \Delta x}{F}.u_{tt}$$

$$\therefore \frac{1}{\Delta x}[u_x(x, x+\Delta x, t) - u_x(x,t)] = \frac{\rho}{F}.u_{tt}$$

Because of $\Delta x \to 0$, then $\lim_{\Delta x \to 0} \frac{1}{\Delta x}[u_x(x, x+\Delta x, t) - u_x(x,t)] = u_{xx}$, It can be written as $u_{xx} = \frac{\rho}{F}.u_{tt}$, It can be simply as $c^2 = \frac{F}{\rho}$. The water wave equation can be written as:

$$u_{tt} - c^2 u_{xx} = 0 \tag{4}$$

The numerical solution of (4) cause the constant variable $k = \mu^2 = \left(\frac{n\pi}{L}\right)^2$, can be written that $\lambda_n = \frac{cn\pi}{L}$. What if λ_n substituted to $T''(t) - c^2T(t) = 0$ will be given:

$$T''(t) - \lambda_n^2 T(t) = 0$$

$$T_n(t) = A_n e^{\lambda_n t} + B_n e^{-\lambda_n t}, \text{ dengan} \begin{cases} e^{\lambda_n t} = \cos(t) + \lambda_n \sin(t) \\ e^{-\lambda_n t} = \cos(t) - \lambda_n \sin(t) \end{cases}$$

so $T_n(t) = A_n\left(\cos(t) + \frac{cn\pi}{L}\sin(t)\right) + B_n\left(\cos(t) - \frac{cn\pi}{L}\sin(t)\right)$, then:
$u_n(x,t) = \left\{(A_n + B_n)\cos(t) + (A_n - B_n)\frac{cn\pi}{L}\sin(t)\right\}\sin\frac{n\pi}{L}x$, with $n = 1,2,\dots,n$ so it can be simply:

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty}\left\{(A_n + B_n)\cos(t) + (A_n - B_n)\frac{cn\pi}{L}\sin(t)\right\}\sin\frac{n\pi}{L}x \tag{5}$$

It can be concluded that (5) is a homogen function on $0 < x < L$. According to result of (5) can be written as:

$$u(x,0) = \sum_{n=1}^{\infty}(A_n + B_n)\sin\frac{n\pi}{L}x = \phi(x) \tag{6}$$

Equation (6) is a Sinus Fourier sequences for $\phi(x)$ so,

$$A_n + B_n = \frac{2}{L}\int_0^L \phi(x)\sin\frac{n\pi x}{L}dx, \text{ dengan } n = 1,2,\dots,n \tag{7}$$

So that,

$$u_t(x,t) = \sum_{n=1}^{\infty}\left\{-(A_n + B_n)\sin(t) + \frac{cn\pi}{L}(A_n - B_n)\cos(t)\right\}\sin\frac{n\pi}{L}x$$

$$u_t(x,0) = \sum_{n=1}^{\infty}\frac{cn\pi}{L}(A_n - B_n)\sin\frac{n\pi}{L}x = \psi(x) \tag{8}$$

Equation (8) is a sinus Fourier sequences for $\psi(x)$ so,

$$\frac{cn\pi}{L}(A_n - B_n) = \frac{2}{L}\int_0^L \psi(x)\sin\frac{n\pi x}{L}dx \tag{9}$$

According to (8) and (9) value of A_n dan B_n is:

*Corresponding Author: Tulus

Sinkron : Jurnal dan Penelitian Teknik Informatika
Volume 7, Number 3, Juli 2023
DOI : https://doi.org/10.33395/sinkron.v8i3.12489
e-ISSN : 2541-2019
p-ISSN : 2541-044X

$$A_n = \left(\frac{1}{L} + \frac{1}{cn\pi}\right)\int_0^L \phi(x)\sin\frac{n\pi x}{L}dx$$
$$B_n = \left(\frac{1}{L} - \frac{1}{cn\pi}\right)\int_0^L \left(\phi(x) - \psi(x)\right)\sin\frac{n\pi x}{L}dx$$

So, the homogen solution to propagation water wave equation is:

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty}\left\{(A_n + B_n)\cos(t) + (A_n - B_n)\frac{cn\pi}{L}\sin(t)\right\}\sin\frac{n\pi}{L}x$$

with,

$$A_n = \left(\frac{1}{L} + \frac{1}{cn\pi}\right)\int_0^L \phi(x)\sin\frac{n\pi x}{L}dx \tag{10}$$
$$B_n = \left(\frac{1}{L} - \frac{1}{cn\pi}\right)\int_0^L \left(\phi(x) - \psi(x)\right)\sin\frac{n\pi x}{L}dx \quad \blacksquare$$

Subsequently, the particle solution of the wave dredging equation given in the form of $P_x(t)$ will be determined. the particulate solution of a dredged sea wave equation is the same as the equation before the dredgment because the factor of $p_x(t)$ is the influence of the outer style of the fixed solution. The particulate solution of a non-homogeneous equation can be determined by the following formula:

$$u(x,t) = \int_0^t \left[\frac{1}{2}\int_{x-c(t-s)}^{x+c(t+s)} f(y,s)dy\right]ds = \frac{1}{2}\int_0^t \int_\Delta P_x(t)dxdt \qquad (11$$

The total equation of the submerged ocean wave equation is the sum of the homogeneous equation with the particular, so that the total solution of the equations (50) and (51) is:

$$u(x,t) = \sum_{n=1}^{\infty}\left\{(A_n + B_n)\cos(t) + (A_n - B_n)\frac{cn\pi}{L}\sin(t)\right\}\sin\frac{n\pi}{L}x + \frac{1}{2c}\int_0^t \int_\Delta P_x(t)dxdt \qquad (12$$

with

$$A_n = \left(\frac{1}{L} + \frac{1}{cn\pi}\right)\int_0^t \phi(x)\sin\frac{n\pi x}{L}dx$$
$$B_n = \left(\frac{1}{L} - \frac{1}{cn\pi}\right)\int_0^L \left(\phi(x) - \psi(x)\right)\sin\frac{n\pi x}{L}dx$$

Experimental Simulation

The necessity for computing during wave model iterations as a result of the meshing of the model with the elements to create a huge element with as many as 185,845 edges led to a significant shift in the value of the wave delay versus time. After waves with respective speeds of 1 m/s, 1,5 m/s, and 2 m/s, the simulation will be used to examine how speed affects the distribution of the value of wave speed impedance. The simulation will then proceed with the wave breaker model being modified in accordance with the length of the wavelength breaker location at distances of 10, 15, and 20 m, respectively.

*Corresponding Author: Tulus

Sinkron : Jurnal dan Penelitian Teknik Informatika
Volume 7, Number 3, Juli 2023
DOI : https://doi.org/10.33395/sinkron.v8i3.12489
e-ISSN : 2541-2019
p-ISSN : 2541-044X

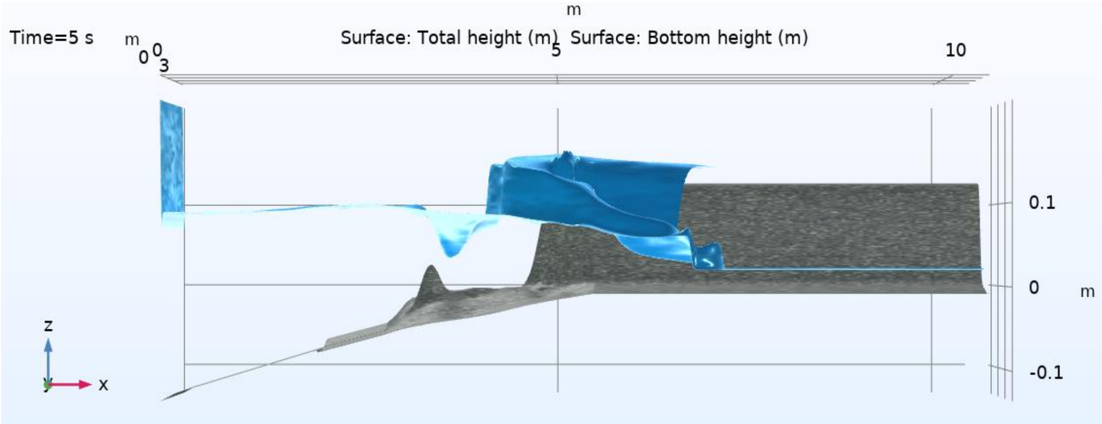

Figure 2. Submerged Permeable Simulation

showing the wave breakdown process with the sinking wave breaker model has permeable properties and is a slowdown on shallow water, a large breakdown speeds each time shown in Figure 3.

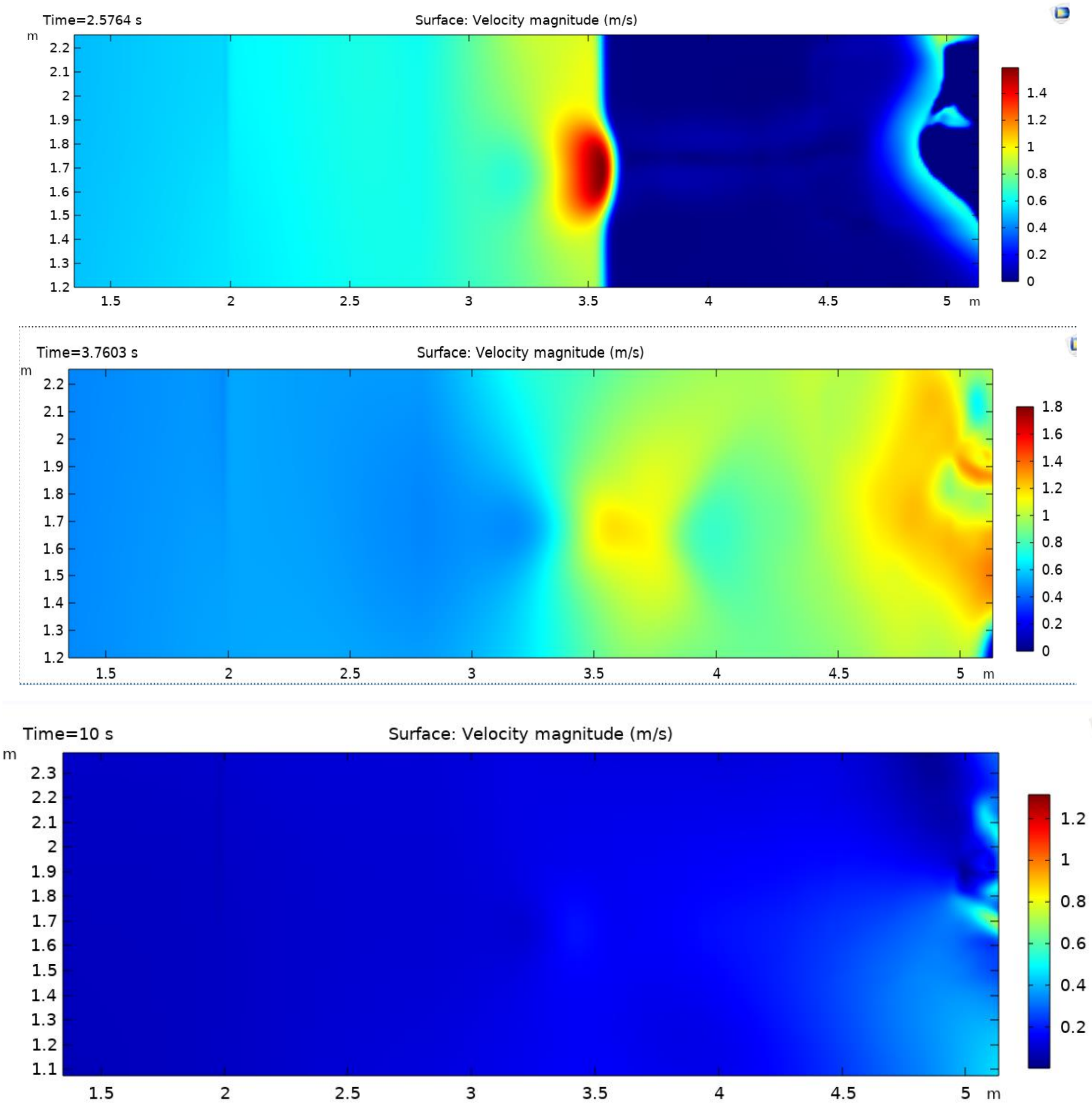

Figure 3. Simulation of the breaking speed of the wave t = [0,10]

*Corresponding Author: Tulus

4,386 iterations of the solution are run for $t = [0,10]$. Water wave convergence indicates that the amplitude on the side of the outlet becomes lower on its simulation, which suggests that the wave breaking process will provide an amplitude breakdown and a minimum wave slowdown speed. Figure 4.7 shows that the inflated water waves show a large highly critical amplitude before and after the wave breakdown process is carried out. To illustrate how the speed spread and wave amplitude varies for different parameter values, a simulation is run on the difference in wave speed of 1 m/s, 1.5 m/s, and 2 m/s.

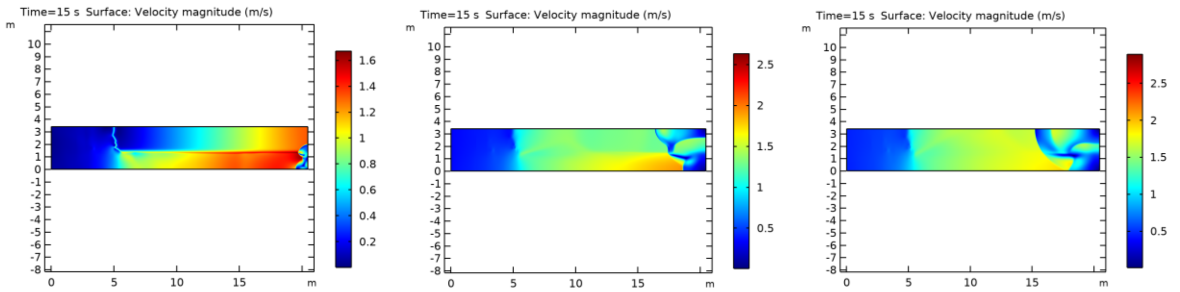

Figure 4. Wave breaker resistance to changes in speed

DISCUSSIONS

In order to limit the large momentum that occurs from the bottom of the shallow area before the wave smashes into the coastal area, the authors propose that the sinking wave breaker provides momentum barriers to the current below the surface. Large-scale simulations of high-wave (λ) variations in the wave breaker position's depth from the ocean's surface were the study's main output. The study also included a simulation of how speed variations affected the waves' enormous amplitudes. A reliability test that progresses against time to a region of stability against the variation of the carried obstacle will be performed on the development of a sinking wave breaker model.

CONCLUSION

Based on the simulations carried out, it can be inferred that the wave breakdown process that moves with wave acceleration delivers a considerable reduction to the change in wave breaker location and wave speed. The research has explained the equation of the inflated wave and the simulated representation displayed on the wave breakdown process, the point that becomes the center of the waves breakdown will give a focused red color indicator meaning there is a change in momentum and potential energy that occurs and then changes the colour of the post-flattering of the sea wave so that the sinking wave breaker is a method to obtain the minimum speed and amplitude values that can be used for coastal engineering.

REFERENCES

Abdul Ghani, F. A., Ramli, M. S., Md Noar, N. A. Z., Mohd Kasim, A. R., & Greenhow, M. (2017). Mathematical modelling of wave impact on floating breakwater. *Journal of Physics: Conference Series*, *890*(1). https://doi.org/10.1088/1742-6596/890/1/012005

Bai, H., Chow, K. W., & Yuen, M. (2022). Exact solutions for the shallow water equations in two spatial dimensions: A model for finite amplitude rogue waves. *Partial Differential Equations in Applied Mathematics*, *5*(December 2021). https://doi.org/10.1016/j.padiff.2022.100360

Bao, Q., Wang, M., Dai, G., Chen, X., Song, Z., & Li, S. (2022). rna Jou. *Swarm and Evolutionary Computation*, *106048*, 101161. https://doi.org/10.1016/j.rinp.2022.106199

Fujima, K. (2006). Effect of a submerged bay-mouth breakwater on tsunami behavior analyzed by 2D/3D hybrid model simulation. *Handbook of Environmental Chemistry, Volume 5: Water Pollution*, *39*(2), 179–193. https://doi.org/10.1007/s11069-006-0022-x

Han, M. M., & Wang, C. M. (2021). Modelling wide perforated breakwater with horizontal slits using

*Corresponding Author: Tulus

Sinkron : Jurnal dan Penelitian Teknik Informatika
Volume 7, Number 3, Juli 2023
DOI : https://doi.org/10.33395/sinkron.v8i3.12489
e-ISSN : 2541-2019
p-ISSN : 2541-044X

Hybrid-BEM method. *Ocean Engineering*, *222*(December 2020), 108630. https://doi.org/10.1016/j.oceaneng.2021.108630

Khater, M. M. A., & Botmart, T. (2022). Unidirectional shallow water wave model; Computational simulations. *Results in Physics*, *42*(September), 106010. https://doi.org/10.1016/j.rinp.2022.106010

Kounadis, G., & Dougalis, V. A. (2020). Galerkin finite element methods for the Shallow Water equations over variable bottom. *Journal of Computational and Applied Mathematics*, *373*(xxxx), 112315. https://doi.org/10.1016/j.cam.2019.06.031

Liu, T. P., & Yu, S. H. (2014). Boundary Wave Propagator for Compressible Navier–Stokes Equations. *Foundations of Computational Mathematics*, *14*(6), 1287–1335. https://doi.org/10.1007/s10208-013-9180-x

Rupali, & Kumar, P. (2021). Mathematical modeling of arbitrary shaped harbor with permeable and impermeable breakwaters using hybrid finite element method. *Ocean Engineering*, *221*(December 2020), 108551. https://doi.org/10.1016/j.oceaneng.2020.108551

Tulus, Khairani, C., Marpaung, T. J., & Suriati. (2019). Computational Analysis of Fluid Behaviour Around Airfoil with Navier-Stokes Equation. *Journal of Physics: Conference Series*, *1376*(1). https://doi.org/10.1088/1742-6596/1376/1/012003

Tulus, Marpaung, J. L., Marpaung, T. J., & Suriati. (2020). Computational analysis of heat transfer in three types of motorcycle exhaust materials. *Journal of Physics: Conference Series*, *1542*(1). https://doi.org/10.1088/1742-6596/1542/1/012034

Tulus, Marpaung, T. J., & Suriati. (2019). Computational Analysis of Water Wheel for Hydro-Electric Power. *Journal of Physics: Conference Series*, *1376*(1). https://doi.org/10.1088/1742-6596/1376/1/012017

Winarta, B., Damarnegara, A., Anwar, N., & Juwono, P. (2018). Analysis of Waikelo Port Breakwater Failure through 2D Wave Model. *Civil and Environmental Science*, *001*(02), 088–095. https://doi.org/10.21776/ub.civense.2018.00102.6

Yu, S., & Huang, L. (2022). Exact solutions of the generalized (2+1)-dimensional shallow water wave equation. *Results in Physics*, *42*(September), 1–6. https://doi.org/10.1016/j.rinp.2022.106020

*Corresponding Author: Tulus