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Direct Search Techniques for Mixed Stochastic Nonlinear Programming Model

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Abstract : Stochastic programming is a methodology utilized for the purpose of achieving optimal planning and decision-making outcomes when faced with uncertain data. The subject of investigation pertains to a stochastic optimization problem wherein the results of stochastic data are not disclosed during runtime, and the optimization of the decision does not necessitate foresight into forthcoming outcomes. This establishes a strong correlation with the imperative need for immediate optimization in uncertain data settings, enabling effective decision-making in the present moment. The present study introduces a novel methodology for achieving global optimization of the model for nonlinear mixed-stochastic programming problem. The present study centers on stochastic problems that are two-staged and entail non-linearities in both the objective function and constraints. The first stage variables are discrete in nature, whereas the second stage variables are a combination of continuous and mixed types. Scenario-based representations are utilized for formulating problems. The fundamental approach to address the non-linear mixed-stochastic programming problem involves converting the model into a deterministic non-linear mixed-count program that is equivalent in form. The feasibility of this proposition stems from the discrete distribution assumption of uncertainty, which can be represented by a limited set of scenarios. The magnitude of the model size will increase significantly due to the quantity of scenarios and time horizons involved. The utilization of filtered probability space in conjunction with data mining techniques will be employed for the purpose of scenario generation. The methodology employed for addressing nonlinear mixed-integer programming problems of significant scale involves elevating the value of a non-basic variable beyond its boundaries in order to compel a basis variable to attain a cumulative value. Subsequently, the problem is simplified by maintaining a constant count variable and modifying it incrementally in discrete intervals to achieve an optimal solution at a global level.

Keywords: Stochastic Programming, Nonlinear Programming, Transportation, Scenario Formation

INTRODUCTION

Presently, the global environment is characterized by a state of heightened uncertainty. Nevertheless, individuals in positions of authority are still required to make decisions despite the prevailing circumstances.

name of corresponding author

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In numerous scenarios, decision problems are expressed in the form of optimization problems. This is because the decision maker seeks to resolve optimization problems that are reliant on parameters that are not yet known. Failure to incorporate this uncertainty into the settlement model may result in suboptimal decision-making or infeasible outcomes.

Uncertainty may manifest in transportation systems due to discrepancies between actual demand for transport resources and demand forecasts, stochastic fluctuations in network link capacity, and stochastic changes in capacity resulting from transport failures.

Stochastic programming pertains to the optimization of decision-making processes in the face of uncertain problem data that may vary across different time periods. The subject of investigation pertains to a stochastic optimization problem, wherein the stochasticity of the data is not disclosed during the runtime, and the optimization decisions are not required to account for future outcomes (non-anticipation). This facilitates a proximate association with the concept of 'real-time' optimization, which pertains to the most advantageous course of action in an environment characterized by incomplete or uncertain data, at the present moment. Assuming the availability of probabilistic data, a viable approach for achieving 'real-time' optimization is to construct a double-stage stochastic program as an operational model. The proposed model aims to supplant the deterministic model by introducing random coefficients or parameters, under the assumption of independent probability distributions of the decision variables.

The class of stochastic optimization problems can be addressed through the utilization of non-linear stochastic programming. These models frequently manifest in practical applications. Numerous natural systems exhibit a non-linear model pattern, which necessitates the use of a non-linear programming approach to determine optimization. The condition of uncertainty appears to have become a ubiquitous factor. It can be asserted that the precise parameters of a system are seldom known. Frequently, it is observed that these parameters are recognized within a spectrum of values or, in certain instances, as a probability distribution. In instances where uncertainty poses a challenge, the application of stochastic programming methodology is warranted.

In certain decision optimization problems, it may be necessary to incorporate a variable that is constrained to take on integer or binary values (i.e., values of either 0 or 1). The non-linear stochastic programming model is classified as stochastic integer non-linear programming when the decision variable is subjected to a counting condition. The present study investigates a stochastic programming model known as stochastic mixed integer non-linear programming (S-MINLP). This model entails variables that are not only restricted to integer values but also allow for continuous values, specifically fractions.

The majority of implementations of the S-MINLP are observed within the domain of process systems engineering. (U. Diwekar, 2005; Sahinidis, 2004) provided a review in the field of process engineering. The utilization of integrated water networks is imperative in the synthesis process (Karuppiah & Grossmann, 2008). Additional applications encompass enterprise networking procedures (Ryu et al., 2004), scheduling and planning associated duties (Jung et al., 2004; Lin et al., 2004), environmental (U. Diwekar, 2005; U. M. Diwekar, 2003; Kheawhom & Hirao, 2004; Widyasari & Mawengkang, 2012) and financial domains (Bastin et al., 2010; Simanjuntak et al., 2006), fisheries (Agustin et al., 2018; Albornoz & Canales, 2006) and transportation network domains (Liu et al., 2009). The problem model of S-MINLP, as suggested in this research, can be expressed in the subsequent manner.

name of corresponding author

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$$\min_x f^1(x) + Q(x)$$
$$g^1(x) = 0,$$
$$h^1(x) \le 0,$$
$$g^1 : R^{nZ_1} \to R^{m_e},$$
$$h^1 : R^{n_1} \to R^{m_i},$$
$$x \in Z_+^{n_1}. \quad (1)$$

Where,

$$Q(x) = E_\xi Q(x, \xi(w))$$
$$Q(x, \xi(w)) = \min_y f^2(y(w), w)$$
$$g^2(x, y(w), w) = 0$$
$$h^2(x, y(w), w) \le 0$$
$$g^2 : R^{n_1+n_2} \times \Omega \to R^{y_e}$$
$$h^2 : R^{n_1+n_2} \times \Omega \to R^{y_i}$$
$$y \in Y \quad (2)$$

The Ω is a mathematical construct that comprises a probability space endowed with a σ-algebra F and associated probability measures. The variable ξ possesses a probability measure and the function $f^1, f^2, g^1, g^2, h^1, h^2$ is non-linear. The variable x denotes the variables pertaining to the first stage, whereas $y(w)$ denotes the variables pertaining to the second stage. The set Y can be expressed as the amalgamation of two distinct subsets, Y_R and Y_Z, wherein Y_R comprises of $Y_R \in R_+^{n_2}$ and Y_Z encompasses $Y_Z \in Z_+^{n_2}$. In the aforementioned model, it is necessary for multiple second-stage variables (indexed by the set Y_Z) to assume integer values.

The salient characteristic of the model for two-stage stochastic programming is the inclusion of a "recourse" action. The decision set has been bifurcated into two distinct groups. Prior to determining the parameters of the problem, a set of decisions must be made. These decisions, referred to as first-stage decisions, are made at the outset of the problem-solving process. Subsequent choices may be made following the disclosure of ambiguity. The determination of recourse is contingent upon both the initial decision made in the first stage and the realization of uncertain parameters. The categorization of the model as a recourse model is attributed to the sequential nature of its events.

The two-stage S-MINLP problem necessitates the consideration of convexity and continuity as crucial factors. This phenomenon can be primarily attributed to the necessity of integer values. In the event that the count variable is solely present in the initial stage, the recourse function retains the same characteristics as in the continuous scenario. If f and h are convex and g is affine for all ξ in the case of continuous nonlinearity, then the problem can be considered as convex. In the event that the count requirement emerges during the second stage, it is noteworthy that the recourse function is typically non-convex for the linear scenario. The complexity of dimensions is contingent upon the quantity of scenarios.

The constraint (2) entails the incorporation of multi-dimensional integration. To address the issue at hand, it is common practice to represent uncertainty using a discrete distribution that closely approximates it. The requirement for precision in modeling leads to an augmented number of dimensions in the optimization

name of corresponding author

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algorithm. The current state of stochastic program modeling and solving methods is constrained by certain limitations, indicating that further development is required.

The utilization of a discrete probability space presupposes that the objective function can be expressed as a finite quantity and that the constraints are duplicated for every constituent of Ω. Assuming that ξ follows a discrete probability distribution where ξ takes the value 1 with a probability of $\Omega = 1, \dots, S$, denoted as $P(\xi = \xi_i) = \pi_i$. The problem can be reformulated in the following manner:

$$\begin{aligned} &\min f^1(x) + \sum_{s=1}^{S} \pi_s f^2(x, y, \xi_s) \\ &g^1(x) = 0 \\ &h^1(x) \le 0 \\ &h_s^2(x, y_s, \xi_s) = 0 \qquad \forall s = 1, \cdots, S \\ &g_s^2(x, y_s, \xi_s) = 0 \qquad \forall s = 1, \cdots, S \\ &x \in Z_+^{n_1}, \quad y_s \in Y_s \qquad \forall s = 1, \cdots, S \\ &g^1 : R^{n_1} \to R^{m_e} \qquad h^1 : R^{n_1} \to R^{m_i} \\ &g^2 : R^{n_1+n_2} \to R^{t_e} \qquad h^2 : R^{n_1+n_2} \to R^{t_i},\ s = 1, \cdots S \end{aligned} \tag{3}$$

The notation π is utilized to denote the likelihood of the occurrence of scenario S. The present formulation can be characterized as a nonlinear integer programming problem with decision variables $n_1 + n_2 s$ and $m_e + m_i + t_e s + t_i s$ nonlinear constraints, which is equivalent in integer.

The recourse function typically exhibits convexity and lower semi-continuity due to the imposed count constraint. The Branch and Bound technique, frequently employed for resolving linear integer programming problems, is not viable for lower semi-continuous constraints due to the infinite number of sub-problems that would be necessary to achieve equivalence between the lower and upper bounds. Hence, there is no assurance of the algorithm's finite termination.

LITERATURE REVIEW

The field of stochastic integer programming (SIP) has recently piqued the interest of researchers. (Haneveld et al., 1996) proposed a two-stage SIP settlement method based on simple integer recourse. Their approach is based on constructing a convex set of second stage value functions due to the special structure of the second stage. When the first and second stage variables are binary, (Laporte & Louveaux, 1993) proposed a decomposition-based approach for SIP. They propose a branch-and-bound method for obtaining the optimality of slicing approximations of non-convex second-stage function values for the first-stage binary solution. (Sen & Higle, 2000) created a decomposition-based algorithm to solve the two-stage SIP problem, focusing on the decomposition of the integer variables that appear in the first and second stages. (Sherali & Fraticelli, 2002) investigated a related approach in which decomposition schemes use linearization-reformulation techniques.

(Carøe & Schultz, 1999) developed a branch and bound algorithm to solve two-stage SIP using (Rockafellar & Wets, 1991)'s scenario decomposition approach. The dual Lagrange lower bound is obtained by dualizing the non-anticipation constraint. The scenario-related dual Lagrange subproblem includes variables and constraints from the first and second stages. This subproblem is more difficult to solve than Benders decomposition. Furthermore, while the dual Lagrange provides a strict limit, its solution necessitates the use of the subgradient method, which presents computational challenges. (Ahmed et al., 2004) also proposed a branch and bound algorithm for two-stage SIP with mixed-integer variables in stage one and pure-integer variables in stage two. In a two-stage SMIP (Stochastic Mixed-Integer Programming), they

name of corresponding author

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used a reformulation that took advantage of the special structure that emerged from the fixed review matrix. (Takriti et al., 1996) proposed the Lagrange relaxation method for solving a two-stage SIP.

(Schultz et al., 1998) proposed a finite scheme for a two-stage stochastic program with a discrete distribution and non-mixed integer variables in the second stage. They discover that only the integer value of the right-hand side parameter is relevant for this problem. This fact is used to identify a countable set in a first-stage variable space that contains the optimal solution, known as the candidate set. To find the best solution, a complete enumeration of the candidate set is performed in its most basic form. To evaluate a set element, a second-stage counting subproblem that corresponds to all possible realizations of uncertain parameters must be solved. Thus, explicit enumeration of all elements is computationally impossible in general. (Simanjuntak et al., 2006) developed a portfolio optimization model by searching for a feasible 'neighborhood' for completing a two-stage PSC.

So far, research in the stochastic mixed integer programming (SMIP) has been limited. (Løkketangen & Woodruff, 1996) solve SMIP with binary variables using a heuristic that combines a progressive hedging algorithm with Tabu search. (Iqbal et al., 2020) also proposed a threshold price heuristic for solving the SMIP in financial optimization, which takes advantage of the structure of the problem with a value at risk. Lagrange relaxation was proposed by (Carøe & Schultz, 1999) for use in a branch and bound algorithm for multiple stage SMIP. The computational results, however, are limited to the two-stage problem. If the probability distribution is discrete and finite, (Schultz & Tiedemann, 2003) investigate the nature of the continuity of the objective function to the first-stage decision and integrate probability measures to further reformulate the two-stage SMIP into a linear mixed integer program. (Lulli & Sen, 2004) proposed the Branch and Price algorithm to solve the multiple stage SMIP problem, which has a unique structure. (Huang & Ahmed, 2009) used specific substructures of the capacity planning problem to develop an efficient approach scheme for solving the multi-stage SIP problem.

(Ahmed et al., 2002) proposed the mean sample approximation (MSA) method to solve stochastic programming with integer recourse. The following is the main idea behind the MSA approach to solving the stochastic program: The sample mean function $N^{-1}\sum_{n=1}^{N} Q(x,\xi^n)$ estimates the expected value function of $E[Q(x,\xi(w))]$ by forming a sample $\xi(w)$ of N random $\xi^1,\ldots,\xi^n$ realization vectors. Approximate sample means obtained:

$$\min_{x\in X}\left\{\hat{g}_N(x) = c^T x + N^{-1}\sum_{n=1}^{N} Q(x,\xi^n)\right\}$$

A deterministic optimization algorithm is then used to solve the stochastic program. This method (and its variants) are also known as the stochastic counterpart method (Rubinstein & Shapiro, 1990) and the path sample optimization method (Plambeck et al., 1996). $\hat{v}_N$ and $\hat{x}_N$ stated the optimal value and optimal solution to the MSA problem, respectively; then v^* and x^* represent the optimal value and optimal solution to the initial problem, respectively.

(Goyal & Ierapetritou, 2007) proposed a "simplicial" approach for S-MINLP settlement. Their approach combines (Goel & Grossmann, 2004; Goyal & Ierapetritou, 2004a, 2004b)'s simplicial-based approach with MSA. The MSA procedure is applied to all linear stochastic subproblems and stochastic mixed linear integer problems at each iteration of the simplicial-based algorithm. However, the nonlinear stochastic program must be solved in this approach, which is also dependent on the number of scenarios.

name of corresponding author

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METHOD

The basic design model can be written in the form,

$$\begin{aligned} &\text{Min} && F(\underline{x}) + \underline{d}^T \underline{y} \\ &\text{Constraint} && f(\underline{x}) + A_1 \underline{y} = \underline{b}_1 \quad (m_1 \text{ baris}) \\ & && A_2 \underline{x} + A_3 \underline{y} = \underline{b}_2 \quad (m_2 \text{ baris}) \\ & && \underline{\ell} \le (\underline{x}, \underline{y})^T \le \underline{u} \end{aligned}$$

The algorithm then works on the main iteration sequence, in which the constraints are linearized at various points along the line $\underline{x}_k$ and the nonlinearity is combined with the objective function and the estimated Lagrange multiplier.

So, it can be written

$$\hat{f}(\underline{x}, \underline{x}_k) = f(\underline{x}_k) + J(\underline{x}_k)(\underline{x} - \underline{x}_k)$$

As a result, in the k-th main iteration, the sub-problems with linear constraints are solved.

That is,

$$\begin{aligned} &\min_{\underline{x}, \underline{y}} \quad L(\underline{x}, \underline{y}, \underline{x}_k, \lambda_k, \rho) = F(\underline{x}) + \underline{d}^T \underline{y} \\ &\qquad - \lambda_k^T (f - \hat{f}) + \tfrac{1}{2} \rho (f - \hat{f})^T (f - \hat{f}) \\ &\text{constraint} \quad \partial_k \underline{x} + A_1 \underline{y} \mp \underline{b}_1 + J_k \underline{x}_k - f(\underline{x}_k) \\ &\qquad A_2 \underline{x} + A_2 \underline{y} = \underline{b}_2 \\ &\qquad \underline{\ell} \le (\underline{x}, \underline{y})^T \le \underline{u} \end{aligned}$$

The objective function is a modified Lagrange expansion, and the penalty parameter ρ hastens convergence of the initial estimation point, which is far from optimal. In order to solve the previous sub-problem, the Lagrange multiplier λ_k is used.

The penalty parameter ρ is reduced to 0 if the main iteration sequence approaches the optimal point (as measured by the relative change in the estimate λ_k and the degree to which the nonlinear constraint $\underline{x}_k$ is satisfied).

The proposed method employs an active constraint strategy, which can be expressed as follows:

$$A\underline{x} = \begin{bmatrix} B & S & N \\ & & I \end{bmatrix} \begin{bmatrix} \underline{x}_B \\ \underline{x}_S \\ \underline{x}_N \end{bmatrix} = \begin{bmatrix} \underline{b} \\ - \\ b_N \end{bmatrix}$$

B Basic vector set

S Superbase vector set

N Nonbasic vector set

I Unit matrix

The non-basic variable $\underline{x}_N$ has reached its limit and will remain so for the next $\Delta\underline{x}$ steps.

name of corresponding author

As a result, it is possible to write it down:

$$B\underline{x}_B + S\underline{x}_S + N\underline{x}_N = \underline{b}$$
$$\underline{x}_N = \underline{b}_N$$

where $\underline{b}_N$ is a combination of upper and lower bounds.

The independent superbasis variable $\underline{x}_S$ moves in any direction and provides a boost to minimize the objective function.

The basis variable $\underline{x}_B$ must follow the following equation:

$$B\underline{x}_B + S\underline{x}_S = 0$$

so, $\Delta\underline{x}$ can be written in terms of changes to the superbase variables as:

$$\Delta\underline{x} = Z\Delta\underline{x}_S$$

with

$$Z = \begin{bmatrix} -B^{-1}S \\ I \\ 0 \end{bmatrix}$$

The matrix Z acts as a "reducing" matrix in this case, multiplying from the left of the gradient vector to form the reduced gradient $\underline{h} = Z^T\underline{g}$ with $\underline{g} = \partial\ell/\partial\underline{x}$. It also multiplies the Hessian matrix of the second partial derivative from left to right to give a Newton-like step in a superbase variable reduced space.

The quasi-Newton R^TR approximation to the reduced Hessian matrix is used in the method's implementation, where R is an upper-triangle matrix. The "sparsity" of the constraints is maintained by storing and updating the Base B matrix's LU factorization.

This factorization implies that Z or B^{-1} are not explicitly stated. The following steps are used to calculate the quasi-Newton $\Delta\underline{x}$ step:

1 Solve $U^TL^T\bar{a} = \underline{g}_B$ for $\bar{a}$ where the gradient vector $\underline{g}$ is partitioned into $(\underline{g}_B, \underline{g}_S, \underline{g}_N)$ with respect to partitions A and $\Delta\underline{x}$

2 Shape $\underline{h} = \underline{g}_S - S^T\bar{a}$

3 Complete $R^TR\Delta\underline{x}_S = -\underline{h}$

4 Solve $LU\Delta\underline{x}_B = -S\Delta\underline{x}_S$

As the search algorithm progresses, the size of the superbase set changes.

If the variable's boundary is crossed, the variable becomes non-basic and is removed from the superbasic (or basis) set.

Meanwhile, if a subspace converges, one or more non-basic variables are used as a superbasis if the 'reduced cost' vector elements associated with $\underline{g}_N - N^T\bar{a}$ are non-zero and marked accordingly.

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The preceding section described a method/algorithm for solving linear and nonlinear stochastic programs. The framework was used to create a method for mixed integer stochastic programming.

Basic Idea

Consider the mixed integer linear programming (MILP) problem:

$$\begin{aligned} \text{Min} \quad & P = \underline{C}^T \underline{x} \\ \text{Constraint} \quad & A\underline{x} \le \underline{b} \\ & \underline{x} \ge 0 \\ & x_j \text{ integer for all } j \in J \end{aligned}$$

MILP's basic feasible vector component, which is solved as a continuous problem, can be written as follows:

$$(x_B)_k = \beta_k - \alpha_{ki}(x_N)_i - \ldots - \alpha_{kj*}(x_N)_{j*} - \ldots - \alpha_{k,n-m}(x_N)_{n-m}$$

Assume $(x_B)_k$ has a count and β_k does not have an integer value; β_k divided into integer components and $\beta_k = [\beta_k] + f_k$ fractions.

If you want to increase $(x_B)_k$ to the nearest integer $([\beta]+1)$, you can do so by increasing a non-base variable, such as $(x_N)_{j^*}$, above the limit as long as α_{kj*} is one of the elements of the negative α_{j*} vector.

If Δ_{j*} is the motion of the non-basic variable $(x_N)_{j^*}$ in such a way that the numeric and scalar values of $(x_B)_k$ are integers, then it can be expressed as:

$$\Delta_{j*} = \frac{1 - f_k}{-\alpha_{kj*}}$$

Other non-basic variables remain unchanged.

So we get $(x_B)_k = [\beta] + 1$ by substituting Δ_{j*} for $(x_N)_{j*}$.

$(x_B)_k$ is now an integer.

The non-base variable clearly plays a significant role in rounding the value of the related basis variable. This fundamental concept is used to solve the nonlinear mixed stochastic counting program on a global scale.

The algorithm of the method

After solving the relaxation problem using the method proposed earlier for the linear stochastic program, the procedure for finding solutions for enumeration areas can be described as follows:

Supposing:

$$x = [x] + f, \qquad 0 \le f < 1$$

continuous solution of the relaxation problem

Step 1. Choose the basis i^* smallest count inefficiency, so that $\delta_{i*} = \min\{f_i, 1 - f_i\}$

Step 2. Perform pricing operations, calculate $v_{i*}^T = \ell_{i*}^T B^{-1}$

name of corresponding author

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Step 3. Compute $\sigma_{ij} = v_{i*}^T a_j$ with j with respect to $\min_j \left\{ \left| \frac{\ell_i}{\sigma_{ij}} \right| \right\}$

I. For non base j at the lower limit

If $\sigma_{ij} < 0$ and $\delta_{i*} = f_i$ calculate $\Delta = \frac{(1-\delta_{i*})}{-\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i*} = 1 - f_i$ calculate $\Delta = \frac{(1-\delta_{i*})}{\sigma_{ij}}$

If $\sigma_{ij} < 0$ and $\delta_{i*} = 1 - f_i$ calculate $\Delta = \frac{\delta_{i*}}{-\sigma_{ij}}$

If $\sigma_{ij} < 0$ and $\delta_{i*} = f_i$ calculate $\Delta = \frac{\delta_{i*}}{\sigma_{ij}}$

II. For non base j at the upper limit

If $\sigma_{ij} < 0$ and $\delta_{i*} = 1 - f_i$ calculate $\Delta = \frac{(1-\delta_{i*})}{-\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i*} = f_i$ calculate $\Delta = \frac{(1-\delta_{i*})}{\sigma_{ij}}$

If $\sigma_{ij} > 0$ and $\delta_{i*} = 1 - f_i$ calculate $\Delta = \frac{\delta_{i*}}{\sigma_{ij}}$

If $\sigma_{ij} < 0$ and $\delta_{i*} = f_i$ calculate $\Delta = \frac{\delta_{i*}}{-\sigma_{ij}}$

Otherwise go to the next non base or superbase j (if any). So column j^* is increased from its lower bound or lowered from its upper bound. If not there go to the next i^*.

Step 4. Calculate

$$\alpha_{j*} = B^{-1} a_{j*}$$

that is, finish $B\alpha_{j*} = a_{j*}$ for α_{j*}

Step 5. The feasibility test has 3 possibilities for the basis variable to remain feasible due to the non-basic variable j^* being folded from its limit.

- if j^* is lower bounded

  Take

  $$A = \underset{i' \neq i^* | \alpha_{ij*} > 0}{\text{Min}} \left\{ \frac{x_{B_{i'}} - \ell_{i'}}{\alpha_{ij*}} \right\}$$

  $$B = \underset{i' \neq i^* | \alpha_{ij*} < 0}{\text{Min}} \left\{ \frac{a_{i'} - x_{B_{i'}}}{-\alpha_{ij*}} \right\}$$

  $C = \Delta$

  The maximum motion of j^* depends on

  $\theta^* = \min(A, B, C)$

- if j^* is upper bound

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Take

$$A' = \min_{i' \neq i^* | \alpha_{ij^*} > 0} \left\{ \frac{x_{B_{i'}} - \ell_{i'}}{-\alpha_{ij^*}} \right\}$$

$$B' = \min_{i' \neq i^* | \alpha_{ij^*} < 0} \left\{ \frac{a_{i'} - x_{B_{i'}}}{\alpha_{ij^*}} \right\}$$

$$C' = \Delta$$

The maximum motion of j^* depends on

$$\theta^* = \min(A', B', C')$$

Step 6. Exchange bases for all 3 possibilities

1. if A or A'
   - $x_{B_{i'}}$ be nonbasic at the lower bound $\ell_{i'}$
   - x_{j^*} Become a base (replace $x_{B_{i'}}$)
   - x_{i^*} fixed basis (not integer).
2. if B or B'
   - $x_{B_{i'}}$ be nonbasic at the upper bound $a_{i'}$
   - x_{j^*} Become a base (replace $x_{B_{i'}}$)
   - x_{i^*} fixed basis (not integer).
3. if C or C'
   - x_{j^*} Become a base (replace x_{i^*})
   - x_{i^*} Become a integer-value superbase.

Repeat from step 1

CONCLUSION

Stochastic programming pertains to the optimization of decision-making processes in situations where there exists uncertainty in problem data across different time periods. The subject of investigation pertains to a stochastic optimization problem characterized by the non-disclosure of the outcomes of random data during the runtime. Additionally, the optimization decisions are not required to anticipate future results, thereby satisfying the condition of non-anticipation. Assuming the availability of probabilistic information, a viable operational framework for optimization can be constructed in the form of a two-stage stochastic program. The proposed model aims to supplant the deterministic model by assuming independent probability distributions for the decision variables, thereby rendering the unknown coefficients or parameters as random. However, in order to address this stochastic programming problem, it is necessary to restructure the model into a deterministic program that is equivalent. Feasibility arises from the assumption that the stochastic parameters possess a probability distribution, thereby enabling the acquisition of the stochastic parameters in the form of scenarios. The deterministic program's dimensional scale is expected to experience a significant increase due to the vast number of scenarios. As a result, the utilization of scenario formation techniques is imperative in order to prevent the occurrence of the aforementioned 'explosion'.

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