

# Optimization Model for Electric Vehicle Routing Problem with Two Charging Options

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Submitted : Jun 21, 2023 | Accepted : Jun 21, 2023 | Published : Jul 1, 2023

**Abstract.** The use of electric vehicles recently is an alternative way to lower pollutants and the emissions of carbon dioxide resulting from the usage of motor vehicle fuel oil. The limited battery capacity of this electric vehicle is a problem for its users. Vehicle routing problem (VRP) is a problem of integer programming and combinatorial optimization that is frequently used in planning and decision-making processes. One application of this issue is to find the best path for delivering items from a corporation to customers. VRP problems are frequently utilized in order to reduce internal expenditure. Model optimization in this paper uses two recharging options, recharging and swapping battery. The purpose of this paper is to present a VRP optimization model for electric vehicle routing problems to find the best route option that minimizes the total of the fees for fixed vehicles, transit, charging, battery changing, and waiting.

**Keywords:** electric vehicle, vehicle routing problem, charging and battery changing, waiting fee.

## INTRODUCTION

The use of electric vehicles recently is an alternative solution to lower pollutants and carbon emissions due to the use of motor vehicle fuel oil. Electric vehicles (EV) apply electric power to move and are suitable for various kinds of transportation require such as public transit, deliveries from food stores to customers' homes, postal and courier companies, and distribution businesses in many industries (Touati-Moungla & Jost, 2012). Until now, logistics companies are still very minimal in using electric vehicles as their logistics vehicles. This is for two reasons. First, the main component of purchasing an electric vehicle is the cost of the battery. Second, the limited battery capacity of electric vehicles makes electric vehicles unsuitable for long-distance deliveries. To attract enthusiasts for EV users, especially in the logistics sector, it is necessary to provide a solution to the problem of using EV in the logistics sector. The distribution of goods or services is an important part of a logistics company (Farooq et al., 2019).

In mathematics, the distribution problem can be solved with the concept of graph theory so that it can be described briefly, because the use of diagrams and symbols or symbols will be easier to understand and easier to solve. The Vehicle Routing Problem (VRP) is a part of the fundamental ideas in graph theory that may be utilized in the solution of distribution issues. Minimizing costs in the distribution process by optimizing routes is frequently referred to as the vehicle routing issue (R. A. Sarker & Newton, 2007). In various planning and decision-making processes, the vehicle routing issue, an optimization by combinatorial and integer-based programming problem, is frequently used to find the best path for delivering commodities from producers to consumers.

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Until now, research on EV is still being carried out. As in 2013, this study discusses an alternative for charging EV batteries, namely Battery Swapping Stations (BSS) and its optimization model. Where BSS can reduce customers' worries about lengthy charge times (M. R. Sarker et al., 2013). The research Electric Vehicle Routing Problem with Charging Time and Variable Travel Time, presents a mathematical method created to solve operational concerns like range restriction and charging demand in the routing problem for electric vehicles with time for charging and variable trip duration (Andani et al., 2022). To establish the routes, the timing of the vehicle departs the depot, and the charging schedule, the model is solved using genetic algorithms (Shao et al., 2017). Different from previous studies, the purpose of the present research is to offer an optimization model for issues involving the routing of electric vehicles that minimizes the total of the expenses associated with stationary vehicles, transit, charging, battery swapping, and waiting cost.

### LITERATURE REVIEW

The basic VRP model is formulated with a dual-index model flow of vehicle flow using a  $(O(n(2)))$  binary variable  $x$  marked which is denoted as use of the line by a courier on the optimal solution or not optimal. The variable  $x_{ij}$  is 1 if the path  $(i, j) \in A$  where ( $A$  is the set of paths arriving at the optimal solution) and  $x_{ij} = 0$  if the path is not traversed (Toth & Vigo, 2002).

$$\min \sum_{i \in U} \sum_{j \in U} C_{ij} x_{ij} \tag{1}$$

$$\sum_{i \in U} x_{ij} = 1, \forall j \in U \tag{2}$$

$$\sum_{j \in U} x_{ij} = 1, \forall i \in U \tag{3}$$

$$\sum_{i \in U} x_{i0} = K \tag{4}$$

$$\sum_{j \in U} x_{0j} = K \tag{5}$$

$$\sum_{i \in U} \sum_{j \in U} x_{ij} \geq r(s), \forall S \subseteq U, S \neq \phi \tag{6}$$

$$x_{ij} \in 0,1 \tag{7}$$

Constraints (2) and (3) relate to the accuracy of the courier who comes and go from where the consumer is. Meanwhile, constraints (4) and (5) depend on  $K$  couriers arriving at the depot. Number requirements sufficient of the constraints is  $2|U|+1$  while the other constraints are mutually related to each other. Capacity used cutting constraint value (6) in the next article CCC (Capacity Cut Constraint), determine the relationship and solutions of capacity. CCC determines that any truncation  $(U|S < S)$  found from consumer sets with path pieces and numbers from path chunk is always greater than or equal to  $r(s)$ . Equality (6) considers CCC by lowering constraints (2), (3), (4) and (5) in the circumstances that may occur.

$$\sum_{i \notin S} \sum_{j \in S} x_{ij} = \sum_{i \in S} \sum_{i \notin S} \sum_{j \in S} x_{ij}, \forall S \in |U|, S \neq \phi \tag{8}$$

The cut in the path between clues is equal to the value of time based on

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$$\sum_{i \in S} \sum_{j \in S} \geq r(U | S), \forall S \in |U|, 0, 0 \in S \quad (9)$$

alternative solutions to the GSEC (Generalize Subtour Elimination Constraint) with model

$$\sum_{i \in S} \sum_{j \notin S} x_{ij} \leq |S| - r(S), \forall S \in |U|, 0, S \neq \phi \quad (10)$$

constraints (6) and (10) allow the addition of customers exponential with the set of  $n$  consumers is possible.

The commercial potential of battery changing has received a lot of attention because it is a relatively new technology. In particular, (M. R. Sarker et al., 2013) provided a BSS adoption business case. By applying a maximum set coverage model to meet the predicted demand, (Frade et al., 2011) pinpoint the position of charging stations. They take into account two different sorts of demand: residential demand at night and commercial demand during the day. In order to optimize social welfare related to both the transportation and power networks, (He et al., 2013) established an algorithm to determine where to best position a specific amount of publicly accessible charging stations.

Similar to this, most of the information currently available on battery changing focuses on where BSSs should be placed. (Mak et al., 2013), for instance, focus on the strategic choice to develop the infrastructure for battery changing. A battery change station placement routing issue was put out by (Yang & Sun, 2015) that figures out the best places for BSSs and the routes that EVs should take to get there. A multiple depot, a different type of electric vehicle location routing issue with time windows is presented by (Paz et al., 2018). In order for lowering travel distance, they examine three potential models. A number of restrictions on battery changing are also listed by (Den Boer et al., 2013).

Recharging stations have been the subject of extensive research, particularly in relation to where they should be located for EV deliveries. For instance, (Frade et al., 2011) utilize a maximum set coverage model to meet the predicted demand to locate recharging stations. This issue has been studied by (Schneider et al., 2014). They introduce the Electric Vehicle Routing Problem with Time Windows (EVRPTW), an EV-specific variant of VRPTW. The electric vehicle battery can be partially charged faster and with a steady amount due to (Keskin & Çatay, 2016). For electric vehicle routing models, some work has been done on the use of battery changing. Model for mixed-integer programming was established by (Chen et al., 2016) for the problem of electric vehicle routing with strict time windows and battery changing stations. We want to consider in our model how battery changing at a station compares to the conventional recharging approach. As a result, when they choose to visit the station, they have both of these options available to them rather than just battery changing as in (Chen et al., 2016).

### PROBLEM DEFINITION

The VRP entails choosing an group of  $K$  vehicle journeys with a minimal total fees, each starting and ending at the depot, every  $C$  clients with service times  $s_i, i \in [1, C]$ . Since a charging station may be used more than once, let  $C = \{1, \dots, N\}$  stand for the collection of charging stations and their multiple batches. Let  $C'$  be any set of vertices which were  $C' = C \cup F$ . The set is subscripted in order to distinguish each instance of the depot. By utilizing  $O$  or  $O'$ . The initial depot and terminating depot, respectively, are indicated by vertex  $O$  and vertex  $O'$ . Therefore,  $C'_O = C' \cup \{O\}$  and  $C'_{O'} = C' \cup \{O'\}$ . The issue may therefore be described on an entire directed graph  $G = (C'_{O,O}, A)$  that has the group of shapes  $A = \{(i, j) | i, j \in C'_{O,O}, A\}$ . Every shape has A road trip time  $t_{ij}$  and the length  $d_{ij}$  associated with it. The battery is depleted at a sustained rate of  $r$ , while the traveling shape uses some of the power from the  $b \cdot d_{ij}$  battery. If the recharging station chooses partial recharging technology and the voltage battery of  $b$  is used to charge the battery, let  $\phi_i$  be the amount of recharged energy. Otherwise, a fully charged battery will be

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substituted for the current one at a cost of  $c_s$  per battery switch. We examine the time wasted consuming is nil when in comparison to the route's travel time since battery changing may be done in a very short amount of time. Each vertex  $i \in C$  has a time windows  $[e_i, l_i]$ , a service time  $S$ , and a positive demand  $D_i$ . The load capacity of an EV is  $W$ , while the battery capacity is  $W$ . A decision variable  $\tau_i, u_i$  and  $v_i$ , respectively, stand for the service beginning time, still-standing cargo level, and the remaining fee level at customer  $i \in C_{O,O'}$ . The time that the EV must wait at the vertex  $i \in N$  is specified as  $\psi_i \cdot \theta_i$ , which is the battery state of charge at the moment of departure from the charging station  $i \in F$ . The binary decision variable  $x_{ij}$  accepts 1 when the shapes  $(i, j)$  is crossed and 0 when it is not. If the BSS is utilized at station  $i \in F$ , let variable  $\gamma_i$  take 1; if not, let it take 0. If the partial recharge option is selected at station  $\lambda_i$ , let variable  $\lambda_i$  take 1; otherwise, let it take 0

## MATHEMATICAL MODEL

The mathematical model below provides guidance for making strategic decisions on where and when to select battery changing and recharging at certain stations. The parameters and variables that are utilized in integer mixed programming will then be introduced.

### Parameters and Variables

$F_0$	total fee
$f_f$	fee fixed per unit of vehicles
$f_t$	fee of travel per unit
$f_r$	fee of recharging per unit
$f_s$	fee for changing every battery
$f_w$	fee per unit of waiting
$d_{ij}$	vertex $i$ 's and $j$ 's distance
$t_{ij}$	vertex $i$ 's and $j$ 's travel time
$r$	battery consumption rate
$b$	rating of battery charge
$D_i$	the client's request $i$
$s_i$	the client $i$ 's service time
$e_i$	the service at vertex $i$ began early
$l_i$	the sevice at vertex $i$ begin late
$W$	capacity of a vehicle
$K$	vehicle battery capacity
$C$	set of client
$F$	a set of charging stations and replicas of them
$O$	beginning depot
$O'$	closing depot
$C_{CF}$	set of clients and charging stations
$C_{COF}$	set of clients, a beginning depot, and recharge stations

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$C_{COF}$	set of clients, closing depot and recharge stations
$C_A$	set of clients, beginning depot, closing depot and recharge stations
$\tau_i$	the vertex $i$ service begin time
$u_i$	the vehicle's departing cargo level's remaining cargo vertex $i$
$\psi_i$	the electric vehicle's battery level at the vertex $i \in C$
$\phi_i$	In terms of the partial recharge technique recharging quantity is chosen in recharging station $i$
$v_i$	after the vehicle departs from station $\forall i \in F$ , its remaining charge level
$\theta_i$	Vehicle wait time at the vertex $i \in F$
$x_{ij}$	when the path of shape $(i,j)$ equals 1; else it equals 0
$\gamma_i$	when selecting the BSS at the station $i \in F$ , equal 1; else it equals 0
$\lambda_i$	when choosing partial recharge at the station $i \in F$ , equal 1; else it equals 0

$$\text{Minimize } F_0 = f_f \sum_{j \in C_{CF}} x_{0j} + f_t \sum_{i,j \in C_{CF}} x_{ij} t_{ij} + f_r \sum_{i \in C} \lambda_i \phi_i + f_w \sum_{i \in C} \theta_i + f_s \sum_{i \in C} \gamma_i \quad (11)$$

$$\text{subject to } \sum_{j \in C_{COF}, i \neq j} x_{ij} = 1, \forall i \in C \quad (12)$$

$$\sum_{j \in C_{COF}, i \neq j} x_{ij} \leq 1, \forall i \in F \quad (13)$$

$$\sum_{i \in C_{COF}, i \neq j} x_{ij} = \sum_{i \in C_{COF}, i \neq j} x_{ij}, \forall j \in C_{CF} \quad (14)$$

$$\tau_i + (t_{ij} + s_i)x_{ij} - l_0(1 - x_{ij}) \leq \tau_j, \forall i \in C_{COF}, \forall j \in C_{COF}, i \neq j \quad (15)$$

$$\tau_i + t_{ij}x_{ij} + g\phi_i\lambda_i - (l_0 + gK)(1 - x_{ij}) \leq \tau_j, \forall i \in F, \forall j \in C_{COF}, i \neq j \quad (16)$$

$$e_j \leq \tau_j \leq j_j, \forall j \in C_{COF} \quad (17)$$

$$0 \leq u_j \leq u_i - D_i x_{ij} + W(1 - x_{ij}), \forall i \in C_{COF}, \forall j \in C_{COF}, i \neq j \quad (18)$$

$$0 \leq u_0 \leq W \quad (19)$$

$$0 \leq c_j \leq c_i - (b \cdot d_{ij})x_{ij} + K(1 - x_{ij}), \forall i \in C, \forall j \in C_{COF}, i \neq j \quad (20)$$

$$0 \leq c_j \leq Y_i - (b \cdot d_{ij})x_{ij} + K(1 - x_{ij}), \forall i \in F, \forall j \in C_{COF}, i \neq j \quad (21)$$

$$Y_i = \lambda_i(c_i + \phi_i) + y_i K, \forall i \in F \quad (22)$$

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$$\gamma_{ij} + \lambda_j \leq 1, \forall j \in F \quad (23)$$

$$\psi_i \leq K, \forall i \in F \quad (24)$$

$$x_{ij} \in \{0,1\}, \forall i \in C_{COF}, \forall j \in C_{CO'F}, i \neq j \quad (25)$$

$$\gamma_j, \lambda_j \in \{0,1\}, \forall j \in F \quad (26)$$

$$\phi_i \geq 0, \forall i \in F \quad (27)$$

## CONCLUSIONS

Electric vehicle routing problems can be constructed in a multi-objective model associated with the vehicle routing problem. Two recharging options in this model are used in this paper. And five objectives function with consideration are to lower the total of fixed vehicle fees, transit fee, charging fee, and battery changing fee and waiting fee.

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