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Integer Linear Programming Model for Multicast Routing to Minimize Link Cost at Wavelengths

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Abstract. Routing is a technique or process carried out in a packet or information delivery system from a source to a destination or a place that requests the information needed from a source. Routing is done in solving a problem and is well-known is unicast routing. Unicast routing can transmit information from a source to a single destination. However, this is ineffective for an infinite number of requests to be served because the unicast routing system only funnels one destination to one source. So that another route is needed that can handle this, this route is called a multicast route. Multicast routing that can serve multiple destinations with only one source so that a collection of destinations to get the same request in the same time and data. This research will discuss multicast routing to minimize link costs at wavelengths using a hierarchical structure. To help provide reduction or savings on the link by offering an integer linear programming model formulation to minimize link costs at wavelengths. An update from previous studies is to use a hierarchical structure with wavelength conversion taking into account the links that are traversed and the requests that will be served by the source to the destination. Using wavelength conversion so that there are no wavelength continuity problems, so that when sending information you get more path choices and you can save on sending information using a link that will be passed from the source to the intermediate node until it reaches the destination node, with this it will find a path more efficiently so that the goal of minimizing link cost at wavelengths on multicast routing is achieved.

Keywords: Multicast routing, Wavelength, Link cost, Hierarchical structure, Integer linear programming

INTRODUCTION

The development of the times has brought life to a system that is easier and faster. The presence of technology today provides many benefits, especially in the fields of mathematics, business, economics and engineering. Present technology can be developed using a mathematical model. One application using

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a mathematical model that is needed and widely applied in everyday life is routing. Routing is often used to connect a community, organization or company in a network in a new, fast and easily accessible way in carrying out work activities to send information to destinations that request information to be served by sources.

The well-known routing is unicast routing, by using unicast routing, information will be channeled from destination to source. However, unicast routing has a drawback, if there are too many requests. For example, there is a group that requests the same information as 50 destinations, so by using unicast, the routing will be carried out one by one from the source to the first destination, from the source to the second destination, and so on. So this routing is less efficient for cases like this (White et al., 1900).

If this case occurs, another type of routing is needed to overcome the above problems, routing that has a good and effective working system for sending with many destinations, namely multicast routing because it has the ability to make savings in shipping so that information is sent quickly to all the destination only sends once and the destination will get the same information from the source (Oliveira et al., 2007).

There are some important components which can be optimized in routing for example resource allocation. Resources can be optimized, namely bandwidth, congestion or delays, and costs on the link so that it is more effective when sending information. During the process of routing or sending information from source to destination, it often experiences congestion or delays, which will prolong the time and will also increase the cost of the link. Delays that occur when sending from one route to another also affect link costs which will be more expensive because you have to find network routes that can be traversed so that information reaches its destination (Oliveira & Pardalos, 2011).

The use of networks that use multicast routing is one of them is a network that uses long wavelengths. The most common problem in networks that use wavelengths is the link cost at that wavelength. So, it must be able to minimize link costs at wavelengths so as not to spend a lot of link costs on the wavelengths that are transmitted to each link that is traversed until it reaches its destination.

Multicast routing on networks that use wavelengths has been discussed by (Le et al., 2015), the author introduces a tree structure where nodes can be visited more than once in wavelength multicast routing. A given transmission mode allows sending information from a source node to multiple destination nodes. Research that is almost the same as (F. Zhou et al., 2009) was also carried out by (Constantinou & Ellinas, 2013; W. Yang et al., 2016; Y. Yang & Wang, 2004; X. Yu et al., 2012) by offering link savings using the ILP model on a tree structure. However, it is still less efficient in saving link costs at wavelengths.

In the research conducted by (Cousin et al., 2010; O. Yu & Cao, 2006), both authors presented ILP formulations in multicast routing co-groups on sparse networks. However, wavelength conversion and hierarchical structure are not discussed in it. So, compared to the two researchers, the research conducted will examine multicast routing, namely the allocation of wavelengths with wavelength conversion which will be presented as a multicast model for hierarchical strictures with an objective function of minimizing link costs at wavelengths.

In addition (Molnár et al., 2019) presented an ILP model for grouping together on multicast routing and spectrum assignment which was also almost the same as carried out by (F. Zhou et al., 2010), the authors studied data center design based on content distribution. However, the study did not consider wavelength conversion.

Based on the background above and the references presented, in this study an integer linear programming mathematical model will be formed on multicast routing to minimize link costs at wavelengths with various wavelength conversions so that there are no wavelength continuity constraints.

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PRELEMINIARIES

Routing

Routing is the process of sending information from source to destination. Real examples of routing in everyday life are mail delivery, telephone calls, train travel, video conferencing, and various routing applications. To do routing or something important to build routing, the initial steps that must be done are as follows:

- Knowing the destination address
- Recognize which routes will be traversed
- Finding routes so that information gets to the destination
- Choose a path or route to get to the intended destination address.

A routing application for example a school student in New Zealand would download a map of Sri Lanka from a local website. Maps will show natural features, their boundaries, flora and fauna, rainfall, languages, and religions. The download required thousands of IP Routing packets to find their way from Sri Lanka to the student's PC located in New Zealand. Just like someone who is going to order a train ticket, to be seen on the cellphone beforehand, routes will be passed. Routing in the network is a series of tasks required to move or send packets from one route to another until they reach their destination. Routing has almost similarities with the transportation system and mail delivery operations (Malhotra, 2002).

Multicast Routing

Multicast is transmission from one sender (source) to many recipients (destination) that are in a certain group, so that each recipient will receive the same packet. A packet destined for a multicast address will be forwarded by the router to the destination that needs the packet. In this way, multicast becomes an efficient way of sending packets from one source to several destinations for several types of communication (Oliveira & Pardalos, 2011).

Routing is the process of selecting the data packet path so that it can get to the destination address. The device used for routing is called a route (router). A route is required if connecting two or more different networks. Suppose there is a packet that will be sent from address X to address Y, so when the packet wants to change addresses, a route is needed to take it to its destination. Network routing that connects more than two nodes at the same time is called multicast routing. A multicast routing network is a collection of nodes, where information can be sent using a multicast routing strategy (Cerulli et al., 2016).

Application of Graph Theory on Multicast Routing

Networks at wavelengths can be modeled by undirected graph $G(V, E, c, W)$, where *V* represents the set of nodes in network *G* and each node in $V \in v$ is one of *MI* and *MC* nodes. *MI* is a set of nodes in a graph or network *G* that does not have optics that can divide the wavelength from the source to the destination through intermediate nodes, while *MC* is the set of nodes in the *G* network that has optics that can share the wavelength from source to the destination through intermediary nodes (Bu'ulolo & Mawengkang, n.d.).

$$V = \{v | v \text{ is } MI \text{ or } v \text{ is } MC\}$$

E represents the link set of graph *G* which corresponds to the fiber links between nodes in the routing network while W represents the set of wavelengths for each link. Each link $e \in E$ is associated with a cost function on the link which is $c(e)$, and a shared group routing session on multicast routing as $ms(s, D)$

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which is a set of multicast structures such as a tree structure from source s to destination group D simultaneously with:

- Wavelength continuity constraint in the absence of wavelength conversion, the same wavelength must be used continuously in all structural connections.
- Different wavelength constraints, the two trees must be defined with different wavelengths unless there is an edge disjoint.
- Separating constraint on wavelength, it is assumed that k is the wavelength structure to be streamed $LS_i(s, D_i)$ is calculated sequentially for $ms(s, D)$ where $i \in [1, k]$, and $1 \leq k \leq |D|$.

Regarding the optimization of network resources, the total cost (the cost of the channel wavelengths consumed per session of the shared group on multicast routing should be minimized. The total cost can be calculated by the sum of the costs of all the simple structures built for $ms(s, D)$ (Y. Zhou & Poo, 2005).

$$c\big(ms(s, D)\big) = \sum_{i=1}^{k} c(LS_i)$$
$$= \sum_{i=1}^{k} \sum_{e \in LS_i} c(e)$$

Integer Linear Programming Formulation

Integer Linear Programming (ILP) is a linear model with the additional requirement that some or all of the decision variables must be integers. The use of integer variables provides additional flexibility in modeling (Graver, 1975). So, integer programming can generally be written as follows:

Min
$$z = \sum_{i=1}^{n} c_i x_i \tag{1}$$
Subject to :
$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad \forall_i \in \{1,2, \dots \dots, m \tag{2}$$
$$x = (x_1, x_2, \dots \dots, x_n) \in Z^n \tag{3}$$

The integer programming model can also be used to solve problems with answers (1 and 0), for this model the variables are limited to 2 decisions represented by variables, for example as follows:

$$X_j = \begin{cases} 1, & For\ decision\ Yes \\ 0, & \text{For decision No} \end{cases} \tag{4}$$

This model is often referred to as the binary integer program model.

METHODOLOGY

The steps to form an integer linear programming mathematical model on multicast routing to minimize link costs at wavelengths with various wavelength conversions can be carried out as follows:

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1. The problem of multicast routing on networks with long wavelengths is that there are many link costs due to choosing the wrong link path. The routing model will be presented using a graph to show the path that will be traversed until the information reaches many destinations. The purpose of this study is to focus on the benefits and performance of using wavelength conversion in WDM networks. In this research, the problem is to find a set of optical structures with minimum resulting overall link cost and number of wavelengths by offering an integer linear programming (ILP) model.
2. Determine the parameters to form a multicast routing model on the wavelength network in order to minimize the overall cost on the link. In this case what is used is the request on the wavelength on the link, the cost (cost) on the link, the wavelength, the set of incoming links, the set of outgoing links, incoming links from intermediate nodes, incoming links from sources to intermediaries, nodes from intermediary to the destination node. And there are also decision variables such as binary variables.
3. Determine the objective function with the aim of minimizing link costs on multicast routing with wavelength networks on optical structures based on the parameters that will be presented later.
4. Determine the constraint function of the multicast routing model on a WDM network in order to minimize the overall cost on the link. In this case there will be some constraints to form a model to support the objective function in minimizing link costs.

RESULT AND DISCUSSION

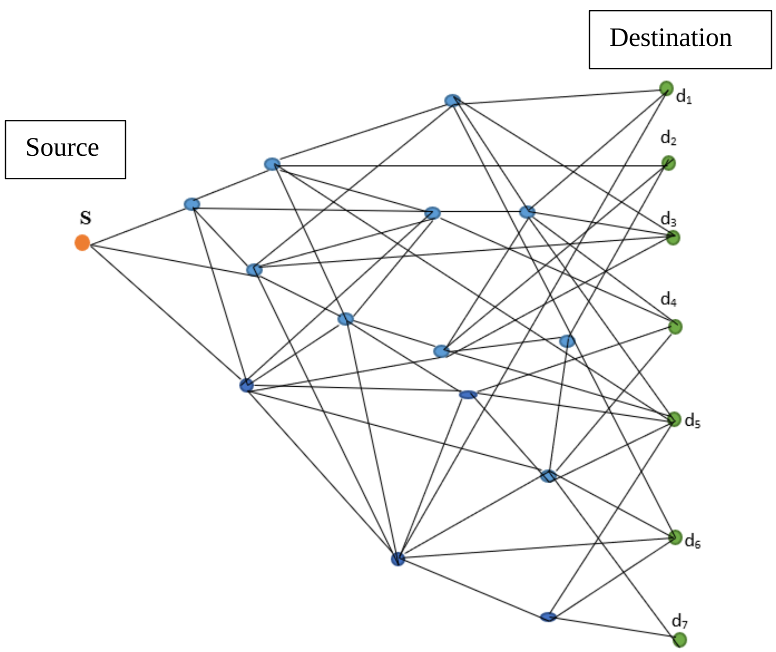

Figure 1. Graph Topology in Multicast Routing

The case in this study assumes that multicast routing on the network has a wavelength conversion in a hierarchical structure. Mathematical model for minimizing link costs at wavelengths with multicast routing. The model will show the source constraints and destination constraints and the equations for the hierarchical structure constraints in the shared group on multicast routing using integer linear

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programming will be shown. The following is an overview of the network using graph theory which will be modeled on the integer linear programming:

The assumptions given to the group shared routing problem in this multicast routing are:

1. A shared group route in multicast routing on a WDM network is a connected graph.
2. All links in the network are routed and the same pool of available wavelengths corresponds to the link that has the lower cost.
3. All linked requests are converted in such a way that requests are processed at one time

Parameters and Notation

Notation and network parameters in the model offered are as follows:

G	:	The multicast routing graph formed on *V* and *E*
V	:	The set of multicast routing nodes on a wavelength network
E	:	Set of multicast routing links/edges on a long-wavelength network
$l_{m,n}$	:	edge / routing links at long wavelengths are connected between node *m* and node *n*
W	:	The set of wavelengths in an optical link group routing together in multicast routing
λ	:	Wavelength $\lambda \in W$
$In_{(m)}$	:	The set of incoming links to node m
$Out_{(m)}$	:	The set of links coming out of node m
$MI_{(G)}$	:	The *G* node set does not have optics that can divide the wavelength emission from the source to the destination through intermediate nodes, including in *V*
$MC_{(G)}$	:	The *G* node set has optics that can divide the wavelength emission from the source to the destination through intermediate nodes, including in *V*
$ms(s, D)$	:	Multicast routing sending requests from the source node to the destination pool
$L_{n,s}(\lambda)$	:	The incoming link from source *s* to node *n* is given a wavelength λ
$C_{m,n}$	:	Overall link cost at node *m* to node *n*

Decision Variables

The decision variables in the multicast routing problem model are described as follows:

$F_{m,n}(\lambda)$: this variable indicates the number of destinations served by multicast routing on the optical link $E_{m,n}$ at wavelength λ.

$$S(\lambda) = \begin{cases} 1 \; if\ the\ wavelength\ \lambda\ is\ used\ in\ the\ optical\ structure\ of\ the\ multicast\ hierarchic \\ 0 \;\; otherwise \end{cases}$$

$$L_{m,n}(\lambda) : \begin{cases} 1 \;\; \text{if request on } ms(s, D) \text{ multicast routing uses wavelength } \lambda \text{ on links } (m, n) \\ 0 \;\; otherwise \end{cases}$$

Model

The purpose of the problem of group routing together in WDM multicast routing is the development of the times and the world of work that requires sending data only by sending using the network, but this is an obstacle because there will be many link costs that will be flowed to get to the

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destination, so this model is used to minimize the link cost of channeled wavelengths built for shared group routing sessions on ms(s,D) multicast routing with the offered model using hierarchical and tree structures in saving link costs but without reducing the packets to be sent or all destinations will get the same packet from the original link that is routed by the source through intermediary routers (from the source node to node n to m) until it arrives at all destinations.

The objective function is defined as follows:
Objective function:

$$Min \sum_{\lambda \in W} \sum_{n \in In(m)} L_{n,m}(\lambda) \times C_{n,m} \tag{4.1}$$

Subject to :

$$\sum_{\lambda \in W} \sum_{n \in Out(s)} F_{s,n}(\lambda) = |D| \tag{4.2}$$

$$\sum_{\lambda \in W} \sum_{n \in In(d)} F_{n,d}(\lambda) = \sum_{\lambda \in W} \sum_{n \in Out(d)} F_{d,n}(\lambda) + 1, \quad \forall d \in D \tag{4.3}$$

$$\sum_{n \in In(d)} F_{n,d}(\lambda) - 1 \leq \sum_{n \in Out(d)} F_{d,n}(\lambda) \leq \sum_{n \in In(d)} F_{n,d}(\lambda), \forall \lambda \in W \tag{4.4}$$

$$\sum_{\lambda \in W} \sum_{n \in In(m)} F_{n,m}(\lambda) = \sum_{\lambda \in W} \sum_{n \in Out(m)} F_{m,n}(\lambda), \quad \forall \lambda \in W \tag{4.5}$$

$$F_{m,n}(\lambda) \geq L_{m,n}(\lambda), \forall m, n \in V, \forall \lambda \in W \tag{4.6}$$

$$F_{m,n}(\lambda) \leq |D| \times L_{m,n}(\lambda), \forall m, n \in V, \forall \lambda \in W \tag{4.7}$$

$$\sum_{\lambda \in W} \sum_{n \in In(s)} L_{n,s}(\lambda) = 0 \tag{4.8}$$

$$1 \leq \sum_{\lambda \in W} \sum_{n \in Out(s)} L_{s,n}(\lambda) \leq |D| \tag{4.9}$$

$$1 \leq \sum_{\lambda \in W} \sum_{n \in In(d)} L_{n,d}(\lambda) \leq |D|, \forall d \in D \tag{4.10}$$

$$S(\lambda) \geq L_{m,n}(\lambda), \forall m, n \in V, \forall \lambda \in W \tag{4.11}$$

$$\sum_{n \in In(m)} L_{n,m}(\lambda) \leq 1, \quad \forall \lambda \in W, \forall m \in MC_{(G)} \tag{4.12}$$

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$$\sum_{n\in Out(m)} L_{m,n}(\lambda) \leq Out_{(m)} \times \sum_{n\in In(m)} L_{n,m}(\lambda),\ \ \forall m \in MC_{(G)} \tag{4.13}$$

$$\sum_{\lambda\in W}\sum_{n\in Out(m)} L_{m,n}(\lambda) \leq Out_{(m)} \times \sum_{\lambda\in W}\sum_{n\in In(m)} L_{n,m}(\lambda), \qquad \forall\lambda \in W, \forall m \in MC_{(G)} \tag{4.14}$$

$$\sum_{n\in Out(m)} L_{m,n}(\lambda) \leq \sum_{n\in In(m)} L_{n,m}\,(\lambda), \forall\lambda \in W, \forall m \in MI_{(G)} \tag{4.15}$$

$$\sum_{n\in Out(m)} L_{m,n}(\lambda) \geq \sum_{n\in In(m)} L_{n,m}(\lambda), \forall m \in V, \tag{4.16}$$

$$\sum_{\lambda\in W}\sum_{n\in Out(m)} L_{m,n}(\lambda) \leq \sum_{\lambda\in W}\sum_{n\in In(m)} L_{n,m}(\lambda)\,, \tag{4.17}$$

$\forall\lambda \in W, \forall m \in MI_{(G)}\,, m \neq s$

$$\sum_{\lambda\in W}\sum_{n\in Out(m)} L_{m,n}(\lambda) \geq \sum_{\lambda\in W}\sum_{n\in In(m)} L_{n,m}(\lambda)\,, \tag{4.18}$$

$\forall m \in V,$

$$\sum_{n\in In(m)} L_{m,n}(\lambda) \leq 1\,, \forall\lambda \in W, \forall m \in MI_{(G)}, m \neq s \tag{4.19}$$

$$\sum_{n\in Out(m)} L_{m,n}(\lambda) \leq 1\,, \forall m \in V, \tag{4.20}$$

$$\sum_{\lambda\in W}\sum_{n\in Out(m)} L_{n,m}(\lambda) = \sum_{\lambda\in W}\sum_{n\in In(m)} L_{m,n}(\lambda)\,, \tag{4.21}$$

$$\forall m \in MI_{(G)}(s \cup D)$$

CONCLUSION

The integer linear programming model to minimize the cost of the wavelength link has been described in the discussion with the existing constraints by using the hierarchical structure of the model offered. In this case the introduction of wavelength conversion could enable paths to be routed using different wavelengths along an optical link.

With the use of this model can eliminate the constraints on the continuity of the wavelength on the network. This allows for a wider choice of paths and therefore the possibility to find more efficient paths that can save or minimize costs on the link.

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