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Mathematical Model for Vehicle Routing and Scheduling with Forward and Reverse Logistics

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Abstract. Companies usually use cross-docking to reduce logistics costs. The product delivery process from suppliers to retailers and vice versa is facilitated by crossdocking facilities. One of important problem in crossdocking is vehicle routes. In this work we discuss about cross-docking problem for vehicle routes which is brought into the form of an integration model. We also present the strategy to handle the forward and reverse logistics. From this strategy we have a NP-hard mathematical model as the result.

Keywords: Vehicle routing,Scheduling with forward,Reverse logistics

INTRODUCTION

In today's competitive environment, has forced them to look for ways to reduce their supply chain costs and total product costs in order to gain more market share, transportation accounts for up to 30% of the total product cost depending on the type of product. Cross-docking is a method that has lately been used by several businesses and investigated by numerous scholars. This strategy seeks to remove or diminish need for ordering and storage. A cross-docking center is a transshipment facility where cargo is temporarily kept nonetheless, some publications consider 24 hours to be the maximum storage time (Sharma & Bojorquez Aispuro, 2020; Van Belle et al., 2012). Vehicle routing is one of the most essential concerns in cross-docking centers. Proper routing decisions can result in lower costs, called efficiency requirements and shorter delivery times, the distribution network with cross-docking into consideration for discussed in this paper because it relates to the problem vehicle routing and transshipment (Farahani et al., 2014).

The transshipment problem studies product transportation through a allocation network in order to determine the best allocation for diverse items in order to reduce overall vehicles and supply costs. The allocation network described above is known as open network with a central cross-docking as the transshipment center, materials travel from the manufacturer to the client via the cross-docking center (Agustina et al., 2014).

Given the time constraints for pickup and delivery, the transshipment capacity problem is limited at each location in the crossdocking facility network. The suggested model's goal function strives to reduce the flow of expenses while meeting all client needs (Chen et al., 2006). They begin with Greedy search and proceed to solve the problem using SA, taboo search (TS), and hybrid algorithms. Transshipment problem in a network of a supplier, a cross-docking facility, and a client, taking both the customer's and the supplier's time constraints into account (Lim et al., 2005). To reduce overall transportation and inventory costs, a polynomial method was used to overcome cross-docking center capacity limits. Similarly, transshipment

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problems were formulated in a number of other studies so that the shipping could be delivered to its destination via direct shipping or through cross-docking centers (Musa et al., 2010).

A mathematical model is proposed to synchronize the pick-up and delivery process via a transshipment center, with the goal function being to reduce total transportation and vehicle operating costs (Lee et al., 2006). Based on the taboo search, a heuristic algorithm was created. To solve the model, a hybrid metaheuristic algorithm with Particle Swarm Optimization and Simulated Annealing features is proposed (Vahdani et al., 2012).

Two assumptions are made by lee's model: to begin with show is vehicles within the pick-up handle set within the transshipment center same time. The moment shows for each client assumed that delivery operations for each client ask may well be transported with more than one vehicle, and more than one vehicle visit each client (Lee et al., 2006). To unravel the issue, a hybrid ant colony-simulated annealing-based approach are presented (Mousavi & Tavakkoli-Moghaddam, 2013).

The problem of vehicle routing is planned to conveyance network by warehouse and crossdocking which expected that client solicitations could be conveyed by means of direct transportation, milk-run examples and point middle people, or a mix of storage and the transshipment procedures (Dondo & Cerdá, 2013). For this situation, the halfway point can act as a stockroom for putting away items for low-request and high-esteem products. Three various of vehicles frameworks can be utilized to convey items from providers to clients comprising of direct delivery, cross-docking, and milk-run conveyance. in this case the model is solved by simulated annealing (Hosseini et al., 2014).

Cross-docking vehicle routing issues, Shipments collected from suppliers and delivered to customers are transported in uniform vehicles. It is assumed that the pick-up and delivery process will only need to be visited once. This problem is solved using a heuristic algorithm (Dondo & Cerdá, 2013). For the first level, the location of the crossdocking center, and the second level, vehicle routing, a two-level mixed integer program was used. Following that, a planning decision is made. (Mousavi & Tavakkoli-Moghaddam, 2013) assumed that all vehicles will arrive at the crossdocking center at the same time and solve the model using hybrid algorithms SA and TS. Considering VRP's decisions in networks with cross-docking capabilities, an approach possible fuzzy hybrid probabilistic programming is developed for the model (Mousavi et al., 2014).

Cross-docking VRP model known as cross-docking retrieval and delivery problem (PDPCD). All arriving vehicles will be expected to halt at the crossdocking center to empty and visit clients after pick-up is total (Santos et al., 2013). for settlement of cost calculation, show will be created. Coordinates the nourishment industry's truck planning issue with vehicle steering to play down general costs, counting warehousing and transportation, as well as the costs of planning and late conveyance of client demands (Agustina et al., 2014).

Forward logistics (FL) vehicles pick up customer demands at intermediate locations (distribution centers, warehouses, or cross-docking centers), unload them there to consolidate and load into departing vehicles, and then distribute them along multiple routes. In reverse logistics (RL), end-of-life products are picked up by vehicle or returned merchandise and transported to the customer's middle location, according to (Dethloff, 2001). Forward and reverse logistics planning, as well as the use of different vehicles for each, result in the need for more vehicles, longer transportation times and costs, and lower vehicle usage.

In this research, VRPSDP or Simultaneous picking and delivery of products is possible in case of vehicle routing problem to find the optimal customer-to-vehicle allocation and optimal route for each vehicle in forward and reverse logistics is the objective function to save the overall cost for the model.

METHODS

This paper employs a three-stage strategy for overcoming forward and reverse logistics. The vehicle leaves the central crossdocking and travels to the supplier to fulfill the customer's request in the SCC section during the early stages of the pick-up process at forward logistics. Tours have been assigned to each vehicle, and the order of suppliers that the vehicle will visit is determined at this stage (Nanayakkara et al., 2022).

Vehicles will load products that have been unloaded and consolidated at the crossdocking center to deliver customer orders and end-of-life products that have been returned to the crossdocking center to be transported to suppliers in the CCC area. Forward and reverse logistics were developed on the ccc section, with simultaneous pick-up and delivery. The third stage for delivery to suppliers of end-of-life products will be temporarily stored at the crossdocking center.

Simultaneous pick-up and delivery of products is possible in the case of vehicle routing for this model. this means that when the vehicle is unloaded at the customer's request, the required product will be returned to the supplier and unloaded at the crossdocking center for delivery to the supplier.

RESULT AND DISCUSSION

Model assumptions for the strategy will be presented as follows:

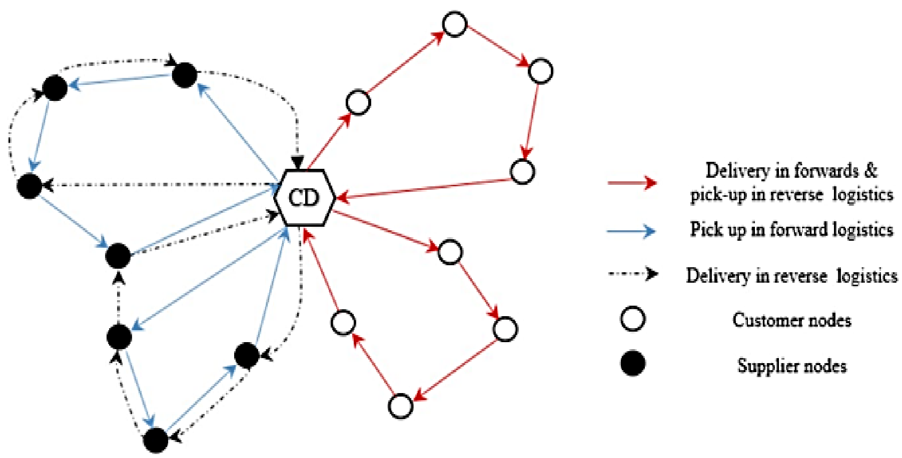

FIG. Schematic of Proposed Network

- There is no product inventory stored at the cross docking center
- Limit capacity of each truck must be considered and must be met
- The Locations are fixed and constant for the cross-docking center, suppliers, and customers.
- The quantity of returned product sent from the customer to the supplier must be sent in forward logistics (FL) and is predetermined and deterministic
- Forward and backward logistics operations using homogeneous vehicles
- Collection of product returns and customer pick-up requests using the same vehicles
- Minimization of total costs on the specified route with forward and reverse logistics no limit on the total tour
- Storage cost is not considered.

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Indices

i	Suppliers
j	Customers
k	Vehicle
CD	Cross-docking center

Sets

S	$\{1,2, \ldots, n_s \vert n_s$ is number of suppliers$\}$
C	$\{1,2, \ldots, n_s \vert n_s$ is number of customers$\}$
V	$\{1,2, \ldots, n_s \vert n_s$ is number of vehicles$\}$

Parameters

c_{ij}	Cost of traveling from i to j
FC	Vehicle fixed costs
t_{ij}	Traveling time from i to j
q_i	Amount of products picked up at i
D_i	Amount of products delivered to i
Cap	Vehicle capacity
St^1	Time service for supplier in the first stage
St^2	Time service for customer in the second stage
St^3	Time service for supplier in the third stage
M^i	A big positive number

Continuous variables

Dt_i^1	Time of supplier vehicle service leave in the first stage
Dt_i^2	Time of customer vehicle service leave in the second stage
Dt_i^3	Time of supplier vehicle service leave in the third stage
At_i^1	Time of supplier vehicle service arrives in the first stage
At_i^2	Time of customer vehicle service arrives in the second stage
At_i^3	Time of supplier vehicle service arrives in the third stage
Lv_k^1	Amount of products loaded in vehicle k leaves in the first stage
Lvo_k^2	Amount of products in vehicle k leaves in the second stage
Lv_i^2	Amount of products in vehicle servicing leaves in the second stage
Lv_k^3	Amount of products in vehicle k leaves in the third stage
u_i^1	Variable for sub-tour elimination, position allocated tour in the first stage
u_i^2	Variable for sub-tour elimination position allocated tour in the second stage
u_i^3	Variable for sub-tour elimination position allocated tour in the third stage

Binary variables

x_{ijk}^1	1 if vehicle k travels in the first stage, otherwise 0
x_{ijk}^2	1 if vehicle k travels in the second stage, otherwise 0
x_{ijk}^3	1 if vehicle k travels in the third stage, otherwise 0

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Optimization Mathematical Model

The mathematical model for cost minimization is as follows:

$$MinZ = \sum_{i\epsilon(SUCD)} \sum_{j\epsilon(SUD)} \sum_{k} c_{ij}x^1_{ijk} + \sum_{i\epsilon(CUCD)} \sum_{j\epsilon(CUCD)} \sum_{k} c_{ij}x^2_{ijk} + \sum_{i\epsilon(SUCD)} \sum_{j\epsilon(SUD)} \sum_{k} c_{ij}x^3_{ijk} + FC.\left(\sum_{j\epsilon S}\sum_{k} x^1_{CDjk} + \sum_{j\epsilon C}\sum_{k} x^2_{CDjk} + \sum_{j\epsilon S}\sum_{k} x^3_{CDjk}\right) \quad (1)$$

Subject To

$$\sum_{i\in(SUCD)} \sum_{K} x^1_{ijk} = 1 \quad \forall j \in S \quad (2)$$

$$\sum_{i\in(CUCD)} \sum_{K} x^2_{ijk} = 1 \quad \forall j \in C \quad (3)$$

$$\sum_{i\in(SUCD)} \sum_{K} x^1_{ijk} = 1 \quad \forall j \in S \quad (4)$$

$$\sum_{i\in(SUCD)} x^1_{ijk} - \sum_{i\in(SUCD)} x^1_{jik} = 0 \quad \forall j \in S, k \in V \quad (5)$$

$$\sum_{i\in(CUCD)} x^2_{ijk} - \sum_{i\in(CUCD)} x^2_{jik} = 0 \quad \forall j \in C, k \in V \quad (6)$$

$$\sum_{i\in(SUCD)} x^3_{ijk} - \sum_{i\in(SUCD)} x^3_{jik} = 0 \quad \forall j \in S, k \in V \quad (7)$$

$$\sum_{j\in S} x^1_{CDj,k} \leq 1 \quad \forall k \in V \quad (8)$$

$$\sum_{j\in C} x^2_{CDj,k} \leq 1 \quad \forall k \in V \quad (9)$$

$$\sum_{j\in S} x^3_{CDj,k} \leq 1 \quad \forall k \in V \quad (10)$$

$$\sum_{j\in S}\sum_{k} x^1_{CDj,k} \geq 1 \quad (11)$$

$$\sum_{j\in C}\sum_{k} x^2_{CDj,k} \geq 1 \quad (12)$$

$$\sum_{j\in S}\sum_{k} x^3_{CDj,k} \geq 1 \quad (13)$$

$$\sum_{i\in(SUCD)}\sum_{k} P_j x^1_{ijk} = Lv^1_k \qquad \forall k \in V \tag{14}$$

$$\sum_{i\in(CUCD)}\sum_{k} D_j x^2_{ijk} = Lvo^1_k \qquad \forall k \in V \tag{15}$$

$$Lv^2_i \geq Lvo^2_k - D_i + P_i - M.\left(1 - x^2_{CD,i,k}\right) \qquad \forall k \in V, i \in C \tag{16}$$

$$Lv^2_j \geq Lv^2_i - D_i + P_i - M.\left(1 - \sum_k x^2_{ijk}\right) \quad \forall i,j \in C \tag{17}$$

$$\sum_{i\in(SUCD)}\sum_{j\in S} D_j x^3_{ijk} = Lv^3_k \qquad \forall k \in V \tag{18}$$

$$Lv^1_k \leq Cap \ \ \forall k \in V \tag{19}$$

$$Lvo^2_k \leq Cap \ \ \forall k \in V \tag{20}$$

$$Lv^2_i \leq Cap \ \ \forall k \in C \tag{21}$$

$$Lv^3_k \leq Cap \ \ \forall k \in V \tag{22}$$

$$Dt^1_{CD} = 0 \tag{23}$$

$$Dt^1_j \geq Dt^1_i + t_{ij} + St^1_i - M.\left(1 - \sum_k x^1_{ijk}\right) \quad \forall\, i \in (S \cup CD), j \in S \tag{24}$$

$$At^1_j \geq Dt^1_i + t_{ij} - M.\left(1 - \sum_k x^1_{ijk}\right) \quad \forall\, i \in (S \cup CD), j \in (S \cup CD) \tag{25}$$

$$Dt^2_{CD} \geq At^1_{CD} + St^1_{CD} \tag{26}$$

$$Dt^2_j \geq Dt^2_i + t_{ij} + St^2_i - M.\left(1 - \sum_k x^2_{ijk}\right) \quad \forall\, i \in (C \cup CD), j \in C \tag{27}$$

$$At^2_j \geq Dt^2_i + t_{ij} - M.\left(1 - \sum_k x^2_{ijk}\right) \quad \forall\, i \in (C \cup CD), j \in (C \cup CD) \tag{28}$$

$$Dt^3_{CD} \geq At^2_{CD} + St^2_{CD} \tag{29}$$

$$Dt^3_j \geq Dt^3_i + t_{ij} + St^3_i - M.\left(1 - \sum_k x^3_{ijk}\right) \quad \forall\, i \in (S \cup CD), j \in S \tag{30}$$

$$At^3_j \geq Dt^3_i + t_{ij} - M.\left(1 - \sum_k x^3_{ijk}\right) \quad \forall\, i \in (S \cup CD), j \in (S \cup CD) \tag{31}$$

$$u^1_j \geq u^1_i + 1 - ns * \left(1 - \sum_k x^1_{ijk}\right) \quad \forall\, i,j \in S \tag{32}$$

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$$u_j^2 \geq u_i^2 + 1 - nc * \left(1 - \sum_k x_{ijk}^2\right) \quad \forall\, i, j \in C \tag{33}$$

$$u_j^3 \geq u_i^3 + 1 - ns * \left(1 - \sum_k x_{ijk}^1\right) \quad \forall\, i, j \in S \tag{34}$$

The objective work of the proposed show, as depicted in equation (1) lessening of vehicle costs and settled vehicle costs of the three stages. A single vehicle must as it was visiting each provider and clients to constraint (2) until (4). A vehicle must exit after going to, unloading, and containing items there, agreeing to constraint (5) to (7). constraint (8) to (10) make beyond any doubt that each vehicle for three stages as it were making a single visit out of the cross-docking office. The constraint (11) through (13) guarantee that hubs ought to be gone by on at slightest one visit amid each of the three stages. The utilized capacity of each vehicle is decided by constraint (14) when arrange one of the visits is total. constraint (15) and (16) show the starting capacity of the vehicle taking off the cross-docking center and the beginning capacity of the vehicle going by the hub after the cross-docking center individually within the visit assigned within the moment organize. Vehicle capacity utilization is calculated in a comparative way to constraint (17) after going by other hubs amid the required visit. The number of products returned which must be conveyed to the supplier during the visit indicated within the third arrange is demonstrated within the constraint (18). The constraint (19) and (22) make sure that the most extreme vehicle capacity is maintained at all times. Vehicles within the introductory organize are required by constraint (23) to take off from the cross-docking center at the begin of the time skyline. Within the to begin with step, constraint (24) and (25) display the times at which a vehicle clears out and returns to a hub, separately. The moment step can begin on the off chance that the primary arrange is wrapped up, concurring to constraint (26). In other words, after wrapping up there to begin with organize of visits, vehicles come at the cross-docking facility and wrap up errands counting emptying, sorting, and stacking. Comparable to constraint (24) and (25), imperatives (27) and (28) moreover apply to the moment arrange. Within the moment arrange the most extreme time required for the vehicle to total the strategy at the crossdocking center is rise to constraint (29). comparable to constraint (24) and (25), constraint (30) and (31) moreover apply to the third arrange. In all three stages, the constraint (32)– (34) are for sub-tour end. Classical VRP and all its variations have a place to NP-hard issues, so they can as it were illuminating small-scale issues ideally with sensible computation time. The demonstrate we propose is additionally NP-hard since it points to create the classical vehicle directing issue. Hence, executing heuristic or metaheuristic calculations appears inescapable. The heuristic calculation utilized for the introductory arrangement in this paper employments recreated strengthening as an optimization algorithm.

CONCLUSION

In this paper companies typically use cross-docking to reduce logistics costs. This cross-docking makes it easier to transfer products from suppliers to retailers and vice versa. Vehicle routes are one example of a cross-docking issue. We discuss the cross-docking problem for vehicle routes in this paper, which is presented in the form of an integration model. In addition, we also present the strategy for dealing with forward and reverse logistics. From this strategy we have a NP-hard mathematical model as the result.

REFERENCES

Agustina, D., Lee, C. K. M., & Piplani, R. (2014). Vehicle scheduling and routing at a cross docking center

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for food supply chains. *International Journal of Production Economics*, *152*, 29–41.
Chen, P., Guo, Y., Lim, A., & Rodrigues, B. (2006). Multiple crossdocks with inventory and time windows. *Computers & Operations Research*, *33*(1), 43–63.
Dethloff, J. (2001). Vehicle routing and reverse logistics: The vehicle routing problem with simultaneous delivery and pick-up: Fahrzeugeinsatzplanung und Redistribution: Tourenplanung mit simultaner Auslieferung und Rückholung. *OR-Spektrum*, *23*, 79–96.
Dondo, R., & Cerdá, J. (2013). A sweep-heuristic based formulation for the vehicle routing problem with cross-docking. *Computers & Chemical Engineering*, *48*, 293–311.
Farahani, R. Z., Rezapour, S., Drezner, T., & Fallah, S. (2014). Competitive supply chain network design: An overview of classifications, models, solution techniques and applications. *Omega*, *45*, 92–118.
Hosseini, S. D., Shirazi, M. A., & Karimi, B. (2014). Cross-docking and milk run logistics in a consolidation network: A hybrid of harmony search and simulated annealing approach. *Journal of Manufacturing Systems*, *33*(4), 567–577.
Lee, Y. H., Jung, J. W., & Lee, K. M. (2006). Vehicle routing scheduling for cross-docking in the supply chain. *Computers & Industrial Engineering*, *51*(2), 247–256.
Lim, A., Miao, Z., Rodrigues, B., & Xu, Z. (2005). Transshipment through crossdocks with inventory and time windows. *Naval Research Logistics (NRL)*, *52*(8), 724–733.
Mousavi, S. M., & Tavakkoli-Moghaddam, R. (2013). A hybrid simulated annealing algorithm for location and routing scheduling problems with cross-docking in the supply chain. *Journal of Manufacturing Systems*, *32*(2), 335–347.
Mousavi, S. M., Vahdani, B., Tavakkoli-Moghaddam, R., & Hashemi, H. (2014). Location of cross-docking centers and vehicle routing scheduling under uncertainty: a fuzzy possibilistic–stochastic programming model. *Applied Mathematical Modelling*, *38*(7–8), 2249–2264.
Musa, R., Arnaout, J.-P., & Jung, H. (2010). Ant colony optimization algorithm to solve for the transportation problem of cross-docking network. *Computers & Industrial Engineering*, *59*(1), 85–92.
Nanayakkara, P. R., Jayalath, M. M., Thibbotuwawa, A., & Perera, H. N. (2022). A circular reverse logistics framework for handling e-commerce returns. *Cleaner Logistics and Supply Chain*, *5*, 100080.
Santos, F. A., Mateus, G. R., & Da Cunha, A. S. (2013). The pickup and delivery problem with cross-docking. *Computers & Operations Research*, *40*(4), 1085–1093.
Sharma, H., & Bojorquez Aispuro, A. (2020). *Evaluating Inventory Risk Pooling Strategy for Multi-Echelon Distribution Network*.
Vahdani, B., Tavakkoli-Moghaddam, R., Zandieh, M., & Razmi, J. (2012). Vehicle routing scheduling using an enhanced hybrid optimization approach. *Journal of Intelligent Manufacturing*, *23*, 759–774.
Van Belle, J., Valckenaers, P., & Cattrysse, D. (2012). Cross-docking: State of the art. *Omega*, *40*(6), 827–846.