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Inventory Model for Order Quantity Optimization with Partial Backlogging on Greater Demand at The Beginning

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Abstract. This article discusses the model of inventory with greater demand at the beginning ($n > 1$) which allows shortages. During the shortage period, it is assumed that there is a backlogged demand, and the remainder is considered lost sales. This research is completed by using the deterministic inventory model method, namely the EOQ model. The result of using the EOQ method is to determine the inventory lot size and length, with the goal of minimizing the total cost of inventory and generating maximum profits related to the inventory model. An numerical example is given to show the use of this model

Keywords: Order quantity,optimization Shortest period, Backlogging

INTRODUCTION

The globalization of markets has led to a significant growth in commerce globally in the twenty first century. To satisfy customer needs, the business manufactures, upgrades, and distributes products. Customers expect products to be produced swiftly and effectively to fulfil demand. Manage production, distribution and maintenance of the commodities to satisfy consumer needs without losing market share, a thorough analysis of when to replace the inventory and how much to order is required. In order for businesses to successfully compete with other businesses, good and proper inventory control is one of their methods (Mathelinea et al., 2019; Tom Jose et al., 2013).

In (Rangkuti, 2007), it is stated that every company, whether the company is a service company or a manufacturing company, always requires inventory. (Heizer & Render, 2004) explained that inventory can have a variety of functions that add to the flexibility of a company's operations. (Siagian, 1987) explained common problems of an inventory model originate from events that are encountered at any time in the business sector, both in the trade sector and in the industrial sector. In (Subagyo et al., 2000) stated that the main problem to be achieved by inventory control is to minimize the total costs incurred by the company.

(Taha, 2013) basically explained inventory analysis with regard to model design to obtain optimal inventory levels by balancing the costs caused by too much inventory with the costs caused by shortages of inventory. (Supranto, 2009) explained the Economic Order Quantity (EOQ) model, namely the number of orders in a certain period must be such that the total ordering costs and storage costs are the same.

Problems that arise in the inventory management system can of course be solved by means of mathematical modeling and optimization techniques (Luis A San-José et al., 2017). Several inventory models have been developed by researchers to deal with inventory control problems. (Luis Augusto San-José et al., 2009) studied a general model for an EOQ inventory system with partial backlogging and linear shortages, (San José et al., 2006) also analyzed the inventory system with exponential partial backlogging, (Sicilia et al., 2012) studied a deterministic supply system with power demand patterns and (Pentico & Drake, 2009) developed a new deterministic EOQ approach with partial backlogging.

Furthermore, the authors are interested in dealing with the problem of inventory control when there is demand when the shortage is met and the rest is lost sales, using the demand pattern index $n > 1$ which can minimize the total cost of inventory so that maximum profits are obtained. This research is a review of the article by (Luis A San-José et al., 2017).

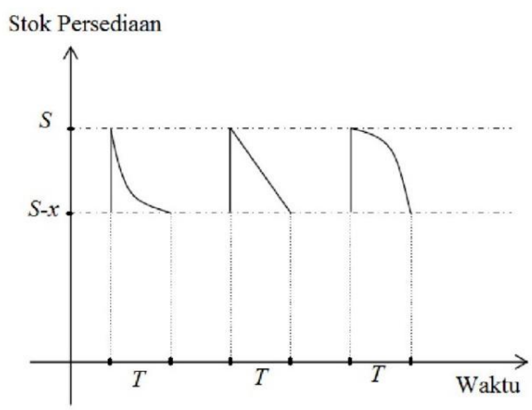
DEMAND PATTERNS

Let x be the total demand during the cycle T and let r be the average demand per cycle i.e., $r = x/T$. The average request is deterministic per unit time. Requests that occur during one cycle are referred to as pattern requests. Thus, the demand pattern over the interval $0 \leq t \leq T$ is assumed to be as follows:

$$D(t) = x\left(\frac{t}{T}\right)^{1/n},$$

With n is the index of demand pattern $0 < n < \infty$. In Figure 1 is explained

FIGURE 1. Demand pattern index



that the maximum number of stocks S in stock during the cycle T for different demand pattern indices is different and the total demand during the cycle is x unit. There are several assumptions that follow the index of demand patterns, including the following:

1. Demand Pattern Index for $n > 1$
   The demand for products $n > 1$ is a greater demand at the beginning of the supply cycle than at the end of the supply cycle or it can be said that customers prefer to buy new items, for example, demand for types of food that have short-term expiry dates, including vegetables, fish, fruit, etc.

2. Demand Pattern Index for $n = 1$
   Product demand for $n = 1$ is the same at the beginning of the cycle as at the end of the cycle or it can be said that the level of demand is constant during the supply cycle, for example, demand for electronic goods, kitchen equipment and household appliances.

3. Demand Pattern Index for $n < 1$
   Product demand for $n < 1$ a demand at the beginning of the cycle is smaller than at the end of the supply cycle or it can be said that demand increases when the amount of inventory is significantly reduced.

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Assumptions and Notations

Given the assumptions and notations that apply to the inventory model that will be used in this study. The assumptions used in the demand pattern index $n > 1$ are as follows:

1. Demand is deterministic at the level of the quantity of goods r per unit time.
2. Supply cycle or recharge cycle T is constant.
3. Unlimited refill levels (orders filled instantly).
4. The time period between the customer order and product delivery is not significant or it can be said that the lead time is equal to zero.
5. Shortages are allowed in stocks and there is unfulfilled demand.
6. Ordering costs and production costs per unit are constant and known.
7. The demand pattern for index $n > 1$ over the interval $(0, t)$ is given as follows:

$$D(t) = x\left(\frac{t}{T}\right)^{1/n} \tag{1}$$

The demand pattern for index $n > 1$ is a greater demand at the beginning of the supply cycle than at the end of the cycle. Furthermore, the notation used in this study is as follows:

τ_1	= the length of the inventory cycle where the net stock is positive (≥ 0)
τ_2	= the length of the inventory cycle when there is a shortage of stock (≥ 0)
T	= inventory cycle $T = \tau_1 + \tau_2$ $(> 0$ decision variables
$I(t)$	= inventory level at time t, with $0 \leq t \leq T$
$D(t)$	= demand pattern over the interval $(0, t)$
x	= total demand during the supply cycle (> 0)
r	= average demand per cycle, that is,$r = x/T$ (> 0)
n	= demand pattern index (> 0)
S	= maximum level on stock, $(\geq 0$ decision variable)
b	= quantity demanded during the shortage period (≥ 0)
β	= parameters when backlog $(0 \leq \beta \leq 1)$
Q	= quantity ordered per cycle, that is, $Q = S + \beta b$
ξ	= the average shortage cost is time dependent, ie $\xi = \omega\beta + \eta(1-\beta)$
ξ_0	= fixed shortfall costs including lost profits, ie $\xi_0 = \omega\beta + (\eta_0 + s - p)(1-\beta)$
η	=unit cost per unit time$goodwill$ (≥ 0)
η_0	= constant *goodwill* cost per unit loss
K	= booking fees per replenishment (> 0)
h	= storage cost per unit time (> 0)
ω	= cost of running outof backorders per unit time (> 0)
p	= acquisition costs(> 0)
s	= selling price per unit$(s \geq p)$
ω_0	= constant cost per unit at the time of backorder(≥ 0)

Figure 2 shows that the supply system is continuous for an unlimited time with a demand pattern index $n > 1$. Inventory level $I(t)$ is a periodic period T and a continuous function on $(0, T)$

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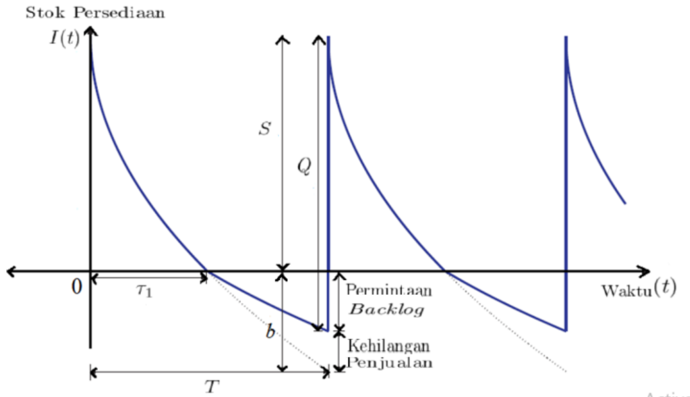
FIGURE 2. Current stock level $\boldsymbol{n > 1}$



Starting from $t = 0$ towards τ_1 per unit time, the level of inventory has decreased significantly due to greater demand at the beginning of the supply cycle $(n > 1)$, i.e. $I(t) = S - D(t)$ then when $t = \tau_1$ the level of inventory runs out $(I(\tau_1) = 0)$, which results $D(\tau_1) = S$, therefore by using the equation (1) obtained the value $\tau_1 = \left(\frac{S}{r}\right)^n T^{1-n}$.

Then when $t = \tau_1$ to T per unit time, the inventory system will experience a shortage, in the shortage period (backlogged) there are β requests that are fulfilled. Thus the inventory level for $t \in [\tau_1, T)$ is $I(t) = \beta(S - D(t))$ and the minimum inventory level is $I(T) = -\beta b = \beta(S - rT)$. In this condition, the cycle repeats at the initial condition as shown in Figure

Furthermore, by considering the assumptions that have been given, revenues and costs in the supply cycle with a demand pattern index $n > 1$ are formulated as follows:

1. Revenue, sQ where Q is the number of orders at shortage βb plus the maximum level of stock S, ie

$$Q = \beta b + S, \tag{2}$$

   And b is the total demand during the inventory cycle rT minus the maximum level in stock S, ie

$$b = rT - S \tag{3}$$

2. Purchase fee pQ
3. Order fee K
4. Storage fee $h\frac{S}{n+1}\left(\frac{S}{rT}\right)^n T$
5. Backorder costs are independent of time $\omega_0\beta(rT - S)$
6. Backorder costs are time dependent $\omega\beta T\left(\frac{n}{n+1}rT + \frac{S}{n+1}\left(\frac{S}{rT}\right)^n - S\right)$

7. The cost of losing a customer is independent of time $\eta_0(1-\beta)(rT-S)$
8. The cost of losing a customer is time dependent $\eta(1-\beta)T\left(\frac{n}{n+1}rT+\frac{S}{n+1}\left(\frac{S}{rT}\right)^n-S\right)$

Then the total profit during the inventory cycle is

$$P(S,T)=(s-p)Q-K-h\frac{S}{n+1}\left(\frac{S}{rT}\right)^nT-\omega_0\beta(rT-S)-\omega\beta T\left(\frac{n}{n+1}rT+\frac{S}{n+1}\left(\frac{S}{rT}\right)^n-S\right)$$
$$-\eta_0(1-\beta)(rT-S)-\eta(1-\beta)T\left(\frac{n}{n+1}rT+\frac{S}{n+1}\left(\frac{S}{rT}\right)^n-S\right), for\ n>1$$

Taking into account that the total demand is equal to the amount ordered per cycle plus lost sales, i.e. $x=Q+(1-\beta)(rT-S)$ earned

$$P(S,T)=(s-p)x-K-h\frac{S}{n+1}\left(\frac{S}{rT}\right)^nT-\xi_0(rT-S)-\xi T\left(\frac{n}{n+1}rT+\frac{S}{n+1}\left(\frac{S}{rT}\right)^n-S\right), for\ n>1$$

Where $\xi_0=\omega_0\beta+(\eta_0+s-p)(1-\beta)$ represents the fixed average shortage cost independent of itme and $\xi=\omega\beta+\eta(1-\beta)$ is the average shortage cost independent on time. The inventory profit per unit of time is given as follows

$$B(S,T)=(s-p)r-C(S,T) \tag{4}$$

By assuming $C(S,T)$ as a function of the total cost of the inventory system when shortages are allowed, the following function $C(S,T)$ is obtained:

$$C(S,T)=\frac{K}{T}+h\frac{S}{n+1}\left(\frac{S}{rT}\right)^n-\xi_0\left(r-\frac{S}{T}\right)-\xi\left(\frac{n}{n+1}rT+\frac{S}{n+1}\left(\frac{S}{rT}\right)^n-S\right) \tag{5}$$

OPTIMAL SOLUTION

This sub-chapter discusses how to obtain the optimal solution for this inventory model, to obtain the optimal solution is to minimize the total inventory cost on the equation (5), that results in the maximum inventory profit per unit of time. By assuming that the first derivative test of the function $C(S,T)$ is then performed S on the following:

$$\frac{d}{dS}C_T(S)=(h+\xi)\left(\frac{S}{rT}\right)^n-\left(\frac{\xi_0}{T}+\xi\right) \tag{6}$$

If $S\geq 0$ there are no shortages, a possible constraint in this inventory cycle is $0\leq S\leq rT$ that some inventories are executed, and some shortages occur. Furthermore, by deriving $C_T(S)$ with respect to S and suppose $f(T)=(\xi T+\xi_0)/[(h+\xi)T]$ and $\tilde{T}=\xi_0/h$ we obtained $C_T(S)$ reaches a minimum when,

$$S^*=\begin{cases} rT & jika\ T\leq\tilde{T}\\ rTf(T)^{1/n} & jika\ T>\tilde{T}, n>1\end{cases} \tag{7}$$

Cost function at optimal value $S^*$for $n > 1$ is as follows:

$$Z(T) = C_T(S^*) = \begin{cases} Z_1(T) = \dfrac{K}{T} + h\dfrac{rT}{n+1} & jika\ T \leq \tilde{T} \\ Z_2(T) = \dfrac{K}{T} + \dfrac{n}{n+1} r(\xi_0 + \xi T)\left(1 - f(T)^{1/n}\right) + \dfrac{1}{n+1}\xi_0 r & jika\ T > \tilde{T}, n > 1 \end{cases} \quad (8)$$

Then from the equation (8)some of the properties of the function $Z(T)$ as follows,

1. Function $Z_1(T)$ has a derivative at $(0, \infty)$ and has a minimum value when,

$$T_0 = \sqrt{\frac{K(n+1)}{hr}}, n > 1 \quad (9)$$

With value

$$Z_1(T_0) = 2\sqrt{\frac{Krh}{(n+1)}}, n > 1 \quad (10)$$

2. Function $Z_2(T)$ has a derivative in $(\tilde{T}, \infty)$. Because, $f'(T) = \xi_0/[(h+\xi)T^2]$ then by deriving the equation $Z_2(T)$ in the equation (8) for $n > 1$ to obtain,

$$\frac{d}{dT} Z_2(T) = \frac{-(n+1)K + n\xi r T^2\left(1 - f(T)^{\frac{1}{n}}\right) + r\xi_0 T f(T)^{1/n}}{(n+1)T^2} \quad (11)$$

By e.g.

$$L(T) = -(n+1)K + n\xi r T^2\left(1 - f(T)^{\frac{1}{n}}\right) + r\xi_0 T f(T)^{1/n} \quad (12)$$

Note that $L(\tilde{T}) = \delta/rh$, Where $\delta = (r\xi_0)^2 - (n+1)Krh$. Furthermore, from the equations (8) and equations (11) obtained:

a. If $\xi_0 = 0$ and $\xi = 0$, then the function $Z_2(T)$ decreases over the interval$(0, \infty)$
b. If $\xi_0 = 0$ and, $\xi \neq 0$ then the function $Z_2(T)$ reaches a minimum when,

$$T_1 = \sqrt{\frac{(n+1)K}{nr\xi\left[1 - \left(\frac{\xi}{h+\xi}\right)^{1/n}\right]}}, n > 1 \quad (13)$$

With value

$$Z_2(T_1) = 2\sqrt{\frac{Knr\xi\left[1 - \left(\frac{\xi}{h+\xi}\right)^{1/n}\right]}{(n+1)}}, n > 1 \quad (14)$$

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c. If $\xi_0 \neq 0$ and $\xi = 0$, then the function $Z_2(T)$ reaches a minimum when,

$$T_2 = \frac{(n+1)K}{r\xi_0}\left(\frac{(n+1)Kh}{r\xi_0^2}\right)^{1/(n-1)}, n > 1 \tag{15}$$

With value

$$Z_2(T_2) = r\xi_0 + \frac{(1-n)r\xi_0}{(n+1)}\left(\frac{r\xi_0^2}{(n+1)Kh}\right)^{1/(n-1)}, n > 1 \tag{16}$$

Based on the description of the properties above, function $L(T)$ plays an important role in solving the optimization problem on the equation (5) . Following are the entries and theorems for the demand pattern index $n > 1$

Lemma 1. Let $L(T)$ the function is given in the equations (12) and $n > 1$

i. If $\xi_0 = 0$ and $\xi = 0$, then $L(T)$ is a constant function and equals $-(n+1)K$
ii. If $\xi = 0$ and $\xi_0 \neq 0$, $L(T)$ are increasing functions on the interval, $[\tilde{T}, \infty)$.
iii. If $\xi \neq 0$ and $\xi_0 = 0$, $L(T)$ are increasing functions on the interval $[\tilde{T}, \infty)$.

Proof. Known function $L(T) = -(n+1)K + n\xi rT^2\left(1 - f(T)^{1/n}\right) + r\xi_0 T f(T)^{1/n}$.

i. If $\xi = 0$ and $\xi_0 = 0$ is obtained the following function $L(T)$

$$L(T) = -(n+1)K + n\xi rT^2\left(1 - f(T)^{\frac{1}{n}}\right) + r\xi_0 T f(T)^{1/n}$$
$$L(T) = -(n+1)K$$

So it proves $L(T)$ to be a constant equal to function $-(n+1)K$

ii. If $\xi = 0$ and $\xi_0 \neq 0$ then the following function $L(T)$ is obtained,

$$L(T) = -(n+1)K + r\xi_0 T f(T)^{\frac{1}{n}}$$
$$L(T) == -(n+1)K + r\xi_0 \tilde{T}^{\frac{1}{n}} T^{\frac{n-1}{n}}$$

The first derivative $L(T)$ when $\xi = 0$ and $\xi_0 \neq 0$ as follows:

$$L'(T) = \frac{r\xi_0 f(T)^{\frac{1}{n}}}{\xi_0}\left(1 - \frac{1}{n}\right)$$

Because $r, \xi_0, f(T)$ is positive and because $n > 1$ therefore $(1 - (1/n))$ is positive, therefore $L'(T) > 0$ and $L(T)$ is an increasing function.

iii. If $\xi \neq 0$ and $\xi_0 = 0$, then $L(T)$ the following function is obtained:

$$L(T) = -(n+1)K + n\xi r\, T^2\left(1 - \left(\frac{\xi}{h+\xi}\right)^{\frac{1}{n}}\right)\xi_0$$

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Then the first derivative $L(T)$ is obtained when $\xi \neq 0$ and $\xi_0 = 0$ as follows

$$L'(T) = 2n\xi rT\left(1 - \left(\frac{\xi}{h+\xi}\right)^{\frac{1}{n}}\right)$$

Because $\xi, h, r, T$is positive and $n > 1$ therefore $1 - \left(\frac{\xi}{h+\xi}\right)^{1/n}$ is also positive, therefore $L'(T) > 0$ and $L(T)$ is an increasing function.

iv. If $\xi \neq 0$ and $\xi_0 \neq 0$, then the function $L(T)$ in the equation (12). Because $n > 1, (1 - f(T)^{1/n})$, and $(1 - (1/n))$ is positive then $L'(T) > 0,\ T > \tilde{T},$ so it $L(T)$ is an increasing function.

Theorem 1. For example $n > 1,\ \delta = (r\xi_0)^2 - (n+1)Krh,\ \tilde{T} = \xi_0/h$ and values T_0, T_1, T_2 are assigned to equation $(9), (13)$and equation (15) , respectively. The value T of the can minimize function $Z(T)$ is given as follow

i. $T^* = T_0$, If $\delta > 0$
ii. $T^* = T_0 = \tilde{T}$, If $\delta = 0$
iii. $T^* = \infty$,If $\xi_0 = \xi = 0$
iv. $T^* = T_r = \arg_{T\in(\tilde{T},\infty)} L(T) = 0$, Otherwise, especially
   a. $T^* = T_1$, if $\xi_0 = 0$ and$\xi \neq 0$
   b. $T^* = T_2$, if $\xi_0 \neq 0$ and $\xi = 0$

Proof. For example, $n > 1,\ \delta = (r\xi_0)^2 - (n+1)Krh, \tilde{T} = \xi_0/h$ and values T_0, T_1, T_2 are assigned to equation $(9), (13)$ and equation (15) respectively

i. If, $\delta > 0$ then it is definite $\xi_0 \neq 0$ and $Z_1(T)$ has a minimum value in $T = T_0 < \tilde{T}$, and $Z_2(T)$ is also an increasing function in $(\tilde{T}, \infty)$ as shown in Lemma 1. So $Z(T)$ it reaches a minimum when $T^* = T_0$
ii. If $\delta = 0$ then $T_0 = \tilde{T}$ and $Z_1(T)$ has a minimum value at $T = T_0 = \tilde{T}$, and is also $Z_2(T)$ an increasing function on $(\tilde{T}, \infty)$as shown in Lemma 1, so that $Z(T)$ it reaches a minimum when $T = T_0 = \tilde{T}$
iii. If $\xi_0 = \xi = 0,$ then $L(T) = -(n+1)K = \delta/rh < 0,$ so $Z_2(T)$ is a decreasing function and $Z(T)$ reaches a minimum at $T^* = \infty$
v. If $\xi \neq 0$ and $\xi_0 \neq 0$, then the function $L(T)$ in the equation (12), $L(T)$ is an increasing function, ξ_0 and ξ are not simultaneously equal to zero and $\delta < 0$, the function $L(T)$ has root T_r in $(\tilde{T}, \infty)$ such that $L(T) < 0$ for $T \in (\tilde{T}, \tilde{T}_r)$ and $L(T) > 0$ for $T \in (T_r, \infty)$, because $L(\tilde{T}) < 0$ and $\lim_{T\to\infty} L(T) = \infty$ (Lemma 1.). As $Z_1(T)$ is strictly decreasing on $(0, \tilde{T})$. We obtain the desired conclusion.

NUMERICAL EXAMPLE

Inventory system for a kind of food product with a demand pattern index 2.5 has an average demand per cycle of a 1000 unit and a demand parameter in the backlog 0.54. The costs incurred by the company (in tens of thousands) include booking fees $Rp500$, acquisition costs $Rp8$, storage costs per unit year $Rp2$, constant shortage costs $Rp0.1$, shortage costs per unit year $Rp3.2$, fixed unit shortage costs including profit loss $Rp1.894$ and constant goodwill costs $Rp2$. Furthermore, the product is sold at a selling price per unit $Rp2$.

From the above statement will be determined the inventory cycle T, maximum stock level S, demand quantity at the time of shortage b and the optimal quantity of goods ordered Q, so that the total cost of the inventory system and the resulting profit will be obtained. The parameters of the inventory system model are known as follows: $n = 2.5$; $\xi = 0$; $\beta = 0.54$; $r = 1000$; $K = Rp500$; $p = Rp8$; $s = Rp2$; $h\ Rp2$; $\omega_0 = Rp0.1$; $\omega = Rp3.2$; $\xi = 0$; $\xi_0 = Rp1.894$; $\eta_0 = Rp2$ and $\eta = 0$

$$\delta = (r\xi_0)^2 - (n+1)Krh = 87236 > 0$$

Furthermore, to calculate the inventory cycle with $\xi = 0, \xi_0 = 1.894 \neq 0$ and $\delta > 0$ use Theorem 1, namely by substituting the parameters above into the equation (9), we obtain the inventory cycle or replenishment cycle T, namely during 0.9354 or 1 year. Next, the value of the maximum level of stock S^* will be calculated using the equation (7) so that the maximum level of stock is obtained, namely 936 units. By using the equation (3)the quantity demanded during the shortage period is obtained and by substituting the parameters into the equation (2), the quantity ordered is 936 units. Furthermore, by substituting the parameters into the equation (10), the total cost of the inventory system is obtained by $Rp1.070$. Profit of the inventory is obtained by substituting parameters into the equation (4), namely equal to $Rp930$. So, the total cost of inventory for 1 year of inventory cycle with the number ordered of 936 units and there is no shortage throughout the cycle is $Rp1.070$, so that the profit is $Rp930$

CONCLUSIONS

An inventory model with an index of demand patterns that allows for shortages, assuming that there is unfulfilled demand and the remainder is lost sales is used to obtain the total inventory cost. The total cost of inventory is obtained using two decision variables, namely the inventory cycle and the maximum stock level. Maximum stock levels and optimum inventory cycles can minimize total inventory costs. The minimum total cost of inventory is useful for maximizing profits, so that profits can avoid losses caused by unfulfilled demand. As a result, this inventory model is useful for assisting companies in making decisions on inventory control issues.$n > 1$

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