

Mathematical Modelling In Logistics Transportation Problems with The Direct Search

Silvi Anggraini Rahman^{1)*}, Herman Mawengkang²⁾, Sutarman³⁾

¹⁾Graduate school of Mathematics, Universitas Sumatera Utara, 20155, Medan, Indonesia

^{2,3)}Department of Mathematics, Universitas Sumatera Utara, Medan, Indonesia

¹⁾silvirahman464@gmail.com ²⁾mawengkang@usu.ac.id ³⁾sutarman@usu.ac.id

Submitted : Jun 25, 2023 | **Accepted** : Jun 25, 2023 | **Published** : Jul 1, 2023

Abstract: Migration of rural communities to cities increases logistics activities in urban areas to meet customer needs because there is a close relationship between economic expansion and usage. Daily fluctuating demand for logistics, uncertain driving times and insufficient parking spaces are some of the factors that link the crisis in urban logistics in urban areas, which directly affects operational costs, the environment and its success or failure. The related steps of modeling optimization have a major impact in making complex transportation and logistics systems competitive with each other. This paper proposes a model optimization to solve transportation problems mathematically. The integer programming model would be suitable for the problems that have been described. the author uses direct search to complete the model.

Keywords: Modelling, Logistic problems, Integer Programming, Direct search

INTRODUCTION

Logistics is often associated with planning, organization, moving, and storing raw and finished goods (Ghani et al., 2013). Likewise organizing resources, the essence of a logistics activity is usually related to the supply of goods with the right type of product, the production and delivery process in the right place, and completion in an efficient time (right product, in the right place, and the right time) better quality accuracy optimization (Roberts, 2007). One of the logistics activities namely; minimizing the total cost of the process of logistics activities and fulfilling the quality of goods in accordance with the wishes of consumers and in accordance with consumer demand. The council of Supply Chain Management Professionals said that in carrying out the logistics process it is necessary, namely, regulate the relationship of goods availability (supply chain) in compiling, implementing, and managing the logistics process and depositing goods, providing information, providing services properly and correctly from the point of origin to the point of destination according to consumer demand (Anderson et al., 2007).

In the current digital era, the types of companies, both on a large, medium and small scale, are building various types of businesses in the culinary, e-commerce, and digital start-up fields so that the emergence of these companies will increase the demand for processed raw materials causing an increase in the production process so that there must be control in the distribution process. (Trentesaux, 2009) stated

Sutarman



that currently distribution has a very vital role in the marketing process which requires the marketing process to run smoothly and facilitate the delivery of goods and services from manufacturers to consumers on time. If the current role of E-logistics is applied to the distribution process, it will be able to control the distribution flow, both orders from customers, incoming and outgoing goods can be controlled at the information center.

There is one system of many systems that is commonly used is to optimize the course of transportation. One solution to the many solutions to problems in transportation is the Vehicle Routing Problems (VRP), which is to compile m sets of vehicle routes with a minimum cost with each vehicle starting and arriving at the depot, each customer is only served once by a vehicle, and the total amount to be purchased and carried does not exceed the capacity of the vehicle (Toth & Vigo, 2002).

In this case the most important thing is that the application of the model for combinatorial logistics transportation in designing logistics operations can raise certain difficulties, because this model does not think about the existing constraints and only answers one question: how to reduce the estimated cost of fuel used to serve a set of buyers? Actually, the cost will depend on the distance traveled, the journey and the load of the vehicle along the corresponding route. Another important part of the delivery cost is the cost of repairing and maintaining the vehicle, which also depends on the total mileage and average load of the vehicle. In addition, combinatorial problems are difficult non-polynomial problems, so they require expansion methods to be solved in general.

LITERATURE REVIEW

Transportation Logistics

Logistics is all activities providing the functions of transportation, storage, assembly, inspection, labeling, packaging, documentation, research and development services for goods and customers. The mission of the logistics system is to distribute products appropriately and optimize value or performance. As one of the logistics works, the planning includes suppliers, location selection, distribution channels, modes of transportation, and the flow of materials (raw, semi-finished and finished products). In this case transportation becomes very important in the distribution of logistics materials (Hübner et al., 2013; Ivanov et al., 2021).

Vehicle Routing Problem

M-TSP is often said to be similar to the vehicle routing problem (VRP), in an area is connected to the one purchased or the consumer, and each vehicle used is considered according to the load. the total amount that one wants to buy on a trip, it is prohibited to exceed the load of transport ordered to go through that rod. (Kallehauge et al., 2005) said m-TSP activities are a type of TSP, which concludes that m-salesmen visit a number of areas, and each area is only visited by exactly one delivery person. Each delivery person starts from a point and at the end of the journey must also return to that point (Abdulkarim & Alshammari, 2015; Jünger et al., 1995).

Vehicle Routing Problem with Time Windows (VRPTW)

Sutarman



This is an Creative Commons License This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License.

VRPTW is an expanded of VRP. In VRPTW, each customer specifies a time limit within which service must be started. VRPTW can be used to model various applications in the field such as bus routes (Bräysy et al., 2009; G. Kim et al., 2015), garbage collectors (Das & Bhattacharyya, 2015; B.-I. Kim et al., 2006), delivery messages (Pilati & Tronconi, 2023; Weigel & Cao, 1999).

Distribution

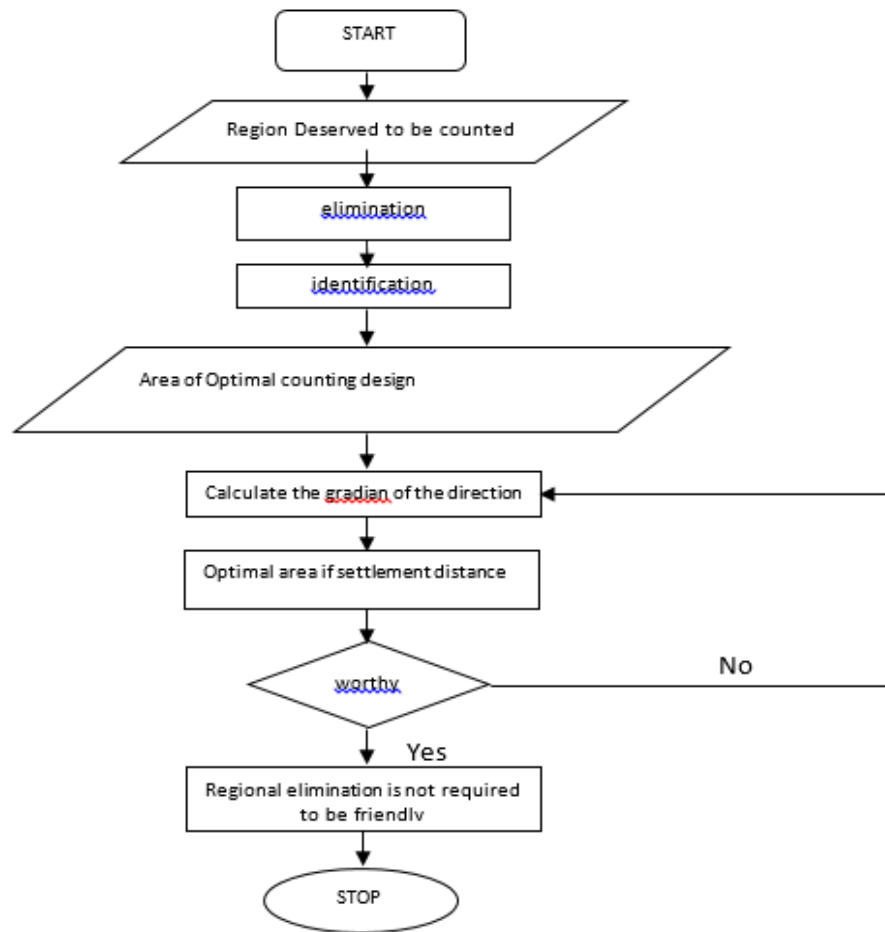
Distribution is a very important aspect in marketing and also implies the manufacture of goods or materials and providing services to meet a need. In the 1990s there were many important studies in manufacturing and factory-based production. So that the Scientific Management Theory emerged, as stated by (Taylor, 2023), advocating standardization of best practices and transforming craft production into mass production, so that in the 20th century researchers began to discuss how alternative ways for conventional production facilities. (Krause et al., 2014) has conducted research that states that products can be more responsive than consumers in terms of product design, so that previous research conducted by (Hayes & Wheelwright, 1985) suggests that there are three manufacturing strategies, namely: size, capacity and location.

Combinatorial Optimization

In the expanding the science of discrete mathematics and computational mathematics the term combinatorial optimization is getting better and can be used in many fields of science the most visible in generating and tidying something without having to count all the events of a series. Combinatorial optimization is also a method used to generate all possible real values of an objective function. The search process can be carried out by compiling the possible values step by step by extending the search algorithm. in producing using one method then choose the best method. means that combinatorial optimization looks for the maximum or minimum value depending on the obstacles described, also this combinatorial optimization is used to solve obstacles that are quite difficult and have a wide range (Korte et al., 2011; Mazyavkina et al., 2021).

RESEARCH METHOD

1. Determine the Eligible Settlement starting point.
2. This part is spent by deleting (eliminating) the part that is not the evidence with the enumeration rules
3. Investigated the part that is able to provide optimal result
4. The equation obtained in stage 2; the tangent is obtained to the part obtained in stage 3
5. Find out how far the part can shift from that point so that is feasibility can be accounted for
6. Investigated the point generated in section 5, is in the feasible section or not.
7. When it's steady, go to step 8, when not repeat to step 4
8. Add a trip from the area generated in stage 5 by cutting the area so that the resulting area in generated in step 2.
9. Stop



RESULT AND DISCUSSION

Problem Description

Explain the sufficient part or part of the way out of the optimization discussion with restraint. If $g(x)$ is a resistance function of the optimization discussion with $g : R^n \rightarrow R$ then the set $\{x : g(x) = b\}$, b a constant is called the sufficient part or the way-out part of the optimization bottleneck.

For combinatorial the function $g(x)$ is define as mapping $g : \{0,1\}^n \rightarrow \{0,1\}$. In order to obtain a feasible area in solving combinatorial optimization problems, there is a feasible region, namely an area that is limited by problem constraints after relaxing the count or binary terms of the variables. For example, the Branch and Bound and Field Slice methods. In general, Metaheuristic methods that have been used by other researchers, such as genetic algorithms, simulation annealing, tabu search, plant propagation, this method does not pay attention to the focus of sufficient parts, but prioritizes the initial results for the previous results which have been described that the group $\{x : g(x) = b\}$ is a sufficient part say a collection $S = \{x : g(x) = b\}$ then when have point $x \in S$ then x is called a feasible settlement point. Indicates the most suitable and result point for example the general form of combinatorial optimization restraint can be describe as follows:

$$\text{Maximize } Z = f(x) \tag{4.1}$$

$$\text{Restraint } g_i(x) = b_i, \quad i = 1, 2, \dots, n$$

$$x \in \{0,1\}$$

if $f(x)$ and $g_i(x) = b_i, (i = 1, 2, \dots, n)$ is a linear function, then problem (4.1) can be written in the explain barries

$$\text{Maximize } Z = C^T X$$

$$\text{Restraint } A x = b \tag{4.2}$$

$$x \in \{0,1\}$$

binary conditions are relaxed then (4.2) can be written as

$$\text{Maximize } Z = C^T X$$

$$\text{Restraint } A x = b \tag{4.3}$$

$$X \geq 0$$

A matrix with $m \times n$ number constraint (m number of rows, n number of columns) can be selected to be the base matrix (B) of size $m \times m$ and an non-basic matrix (N) of size ($m \times (n-m)$)

So, it can be written

$$A = [B \quad N] \tag{4.4}$$

The analog variable x can be selected correspondingly to the vector X_B as a basic variable and X_N non basic variable

Now expression $Ax = b$

$$\text{To be } [B \quad N] \begin{bmatrix} X_B \\ X_N \end{bmatrix} = b \tag{4.5} \quad \text{time}$$

$$B X_B + N X_N = b \tag{4.6}$$

$$\text{or } B X_B = b - N X_N$$

because matrix B adalah basic matrix, then this matrix has invers (B^{-1})

Multiply from the left (4.6) with B^{-1} have

$$B^{-1} B X_B = B^{-1} b - B^{-1} N X_N \tag{4.7}$$

$$I X_B = B^{-1} b - B^{-1} N X_N$$

with I unit matrix.

$$\text{then } X_B = B^{-1} b - B^{-1} N X_N \tag{4.8}$$

value X_n is 0, from the non-negative condition $x \geq 0$

So that a feasible settlement point is obtained (PL) as

$$X_B = \beta \tag{4.9}$$

$$\text{with } \beta = B^{-1} b$$

Optimalisation Calibrating

After generating enough parts from (Equation 4.9), this parts needs to be calibrated whether it is an optimal problem (PL).

See the shift function (PL)

$$Z = C^T X$$

Vector C and X are divided according to the matrix basis (B) and baseless (N)

Then $Z = [C_B \quad C_N] \begin{bmatrix} X_B \\ X_N \end{bmatrix}$ (4.10)

Multiply, obtained

$$Z = C_B X_B + C_N X_N$$
 (4.11)

Substitution equity (5) for X_B , obtained

$$Z = C_B (B^{-1} b - B^{-1} N X_N) + C_N X_N$$
 (4.12)

$$Z = C_B B^{-1} b - C_B B^{-1} N X_N + C_N X_N$$
 (4.13)

$$Z = C_B B^{-1} b - (C_B B^{-1} N - C_N) X_N$$
 (4.14)

supposed $Z_j = C_B B^{-1} N, \forall j \in N$

than $Z_j - C_j = C_B B^{-1} N - C_N, \forall j \in N$ (4.15)

If $Z_j - C_j < 0$ (negative) and the X_N vector set is increased from its limit of 0, it turns out that the value of Z will increase for the problem (PL), which means that the value of the objective function can still increase. So, in other words that the XB point that has been obtained has not resulted in a maximum settlement.

However If $Z_j - C_j < 0$, X_N is increased from its limit of 0 which will result in the Z value decreasing (shrink) or remaining the same, meaning that the XB the resulting part is already the most optimal part, therefore the maximum condition for the problem (PL) has been obtained, namely:

$$Z_j - C_j \geq 0 \text{ "}\forall j \text{ (non-base)"}$$
 (4.16)

The feasible point for CO, that is the feasible point value XB stated by Equation (4.8) is for the problem (PL). Now consider problem (4.2)

The constraint $x \in \{0,1\}$ can be expressed in the form:

$$x \geq 0$$

$$x \leq 1$$

So problem (4.2) can be written

Maximize $Z = C^T X$

Restraint $Ax = b$

$$x \geq 0$$

$$x \leq 1$$

Sutarman



Looking at the problem (PL), the feasible point value also takes the from equation (4.8) i.e.

$$X_B = B^{-1} b - B^{-1} N X_N$$

$$X_B = \beta - \alpha X_N \quad (4.17)$$

with $\beta = B^{-1} b$ and $\alpha = B^{-1} N X_N$

if the problem (PL) X_N has a value of 0 because, in other words the value of the largest variable X_N is at its boundary point, which is 0

In combinatorial problem if the problem (PL) X_N has a value of 0 because, in other words the value of the largest variable X_N is at its boundary point, which is 0.

In combinatorial problems (POL) there are

$$x \geq 0$$

$$x \leq 1$$

So, in the case the value of the non-base X_N variable is 0 or 1, pay attention again to equation (4) it can be seen that the value of the base X_N variable has a value of 0 or 1, meaning that the value of x_n is binary. If all parts of the β vector have a value of 0 or 1, it means that sufficient results for inhibition (PO) have been produced. If the components of the β vector are still there which are not 0 or 1, it means that a sufficient results for inhibition (PO) has not been obtained.

CONCLUSION

1. The proposed model can solve combinatorial optimization problems quickly and produce optimal solutions.
2. By using extension of the metaheuristic method approach, combinatorial optimization results are obtained easily, by determining sufficient sections that are exposed to analytic and exact exposures, so that variables are produced from sufficient sections.
3. The expansion model by taking the metaheuristic method as the tool used is essentially an approximation method, an optimization approach, only with mathematical testing and validation so that the best results are obtained.

REFERENCE

- Abdulkarim, H. A., & Alshammari, I. F. (2015). Comparison of algorithms for solving traveling salesman problem. *International Journal of Engineering and Advanced Technology*, 4(6), 76–79.
- Anderson, D. L., Britt, F. F., & Favre, D. J. (2007). The 7 principles of supply chain management. *Supply Chain Management Review*, 11(3), 41–46.
- Bräysy, O., Dullaert, W., & Nakari, P. (2009). The potential of optimization in communal routing problems: case studies from Finland. *Journal of Transport Geography*, 17(6), 484–490.
- Das, S., & Bhattacharyya, B. K. (2015). Optimization of municipal solid waste collection and transportation routes. *Waste Management*, 43, 9–18.

- Ghiani, G., Laporte, G., & Musmanno, R. (2013). Introduction to Logistics Systems Management. In *Introduction to Logistics Systems Management*. <https://doi.org/10.1002/9781118492185>
- Hayes, R. H., & Wheelwright, S. C. (1985). *Restoring our Competitive Edge: Competing Through Manufacturing*. John Wiley & Sons.
- Hübner, A. H., Kuhn, H., & Sternbeck, M. G. (2013). Demand and supply chain planning in grocery retail: an operations planning framework. *International Journal of Retail & Distribution Management*.
- Ivanov, D., Tsipoulanidis, A., & Schönberger, J. (2021). *Global supply chain and operations management*. Springer.
- Jünger, M., Reinelt, G., & Rinaldi, G. (1995). The traveling salesman problem. *Handbooks in Operations Research and Management Science*, 7, 225–330.
- Kallehauge, B., Larsen, J., Madsen, O. B. G., & Solomon, M. M. (2005). *Vehicle routing problem with time windows*. Springer.
- Kim, B.-I., Kim, S., & Sahoo, S. (2006). Waste collection vehicle routing problem with time windows. *Computers & Operations Research*, 33(12), 3624–3642.
- Kim, G., Ong, Y.-S., Heng, C. K., Tan, P. S., & Zhang, N. A. (2015). City vehicle routing problem (city VRP): A review. *IEEE Transactions on Intelligent Transportation Systems*, 16(4), 1654–1666.
- Korte, B. H., Vygen, J., Korte, B., & Vygen, J. (2011). *Combinatorial optimization* (Vol. 1). Springer.
- Krause, D., Youngdahl, W., & Ramaswamy, K. (2014). Manufacturing - Still a missing link? In *Journal of Operations Management*. <https://doi.org/10.1016/j.jom.2014.09.001>
- Mazyavkina, N., Sviridov, S., Ivanov, S., & Burnaev, E. (2021). Reinforcement learning for combinatorial optimization: A survey. *Computers & Operations Research*, 134, 105400.
- Pilati, F., & Tronconi, R. (2023). Multi-objective optimisation for sustainable few-to-many pickup and delivery vehicle routing problem. *International Journal of Production Research*, 1–30.
- Roberts, T. L. (2007). Right product, right rate, right time and right place... the foundation of best management practices for fertilizer. *Fertilizer Best Management Practices*, 29, 1–8.
- Taylor, F. W. (2023). the Rise of Scientific Management. *The Quantified Worker: Law and Technology in the Modern Workplace*, 9.
- Toth, P., & Vigo, D. (2002). An overview of vehicle routing problems. *The Vehicle Routing Problem*, 1–26.
- Trentesaux, D. (2009). Distributed control of production systems. *Engineering Applications of Artificial Intelligence*, 22(7), 971–978.
- Weigel, D., & Cao, B. (1999). Applying GIS and OR techniques to solve Sears technician-dispatching and home delivery problems. *Interfaces*, 29(1), 112–130.