

Enhancing Vehicle Routing Efficiency through Branch and Bound and Heuristic Methods

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Abstract: The Vehicle Routing Problem (VRP) is a critical challenge in logistics, impacting delivery efficiency and costs. Traditional VRP solutions often fail to address real-world dynamics such as fluctuating traffic conditions and varying customer demands. This research proposes a novel VRP model integrating real-time data to enhance route optimization. By combining the precision of the Branch and Bound (B&B) approach with the flexibility of heuristics like Genetic Algorithms and Simulated Annealing, the hybrid method dynamically adjusts routes based on live traffic and demand updates. The objective is to reduce operational costs and improve logistical performance. The hybrid model's effectiveness is validated through comparative analysis with traditional VRP solutions, demonstrating significant improvements in cost reduction, fuel consumption, vehicle wear and tear, and customer satisfaction due to timely deliveries. These advancements highlight the potential of real-time data integration and advanced optimization techniques in providing robust solutions for modern logistics challenges. Future research should focus on incorporating more advanced data sources and testing the model in various real-world scenarios to further enhance its practicality and performance, ensuring businesses remain competitive in a dynamic market. This study underscores the importance of continuous innovation in VRP solutions to achieve sustainable, efficient, and customer-centric logistics operations.

Keywords: Vehicle Routing Problem; Branch and Bound; heuristic techniques; route optimization; supply chain management

INTRODUCTION

The Vehicle Routing Problem (VRP) poses a major obstacle in the fields of logistics and supply chain management, significantly influencing delivery effectiveness and expenses. Essentially, the VRP focuses on determining the most efficient routes for a fleet of vehicles tasked with delivering goods to numerous customers (Braekers et al., 2016). This problem is more complex than simply finding the shortest path; it requires balancing constraints such as vehicle capacities, delivery windows, and total distance traveled. The main goal is to minimize operational costs while meeting customer demands and logistical constraints (Laporte, 2009).

Addressing the VRP effectively can result in significant savings and enhanced service levels. The bottom-line benefits immediately from efficient routing since it lowers fuel consumption, vehicle maintenance, and labor costs. Timely and trustworthy deliveries are also guaranteed, which increases client satisfaction. Delivery route optimization is a key differentiator for businesses in today's competitive industry (Cordeau et al., 2007).

The VRP becomes more complex as the size of the fleet and the number of delivery points increase. Advanced algorithms and computational methods, including heuristics (Laporte & Semet, 2002; Prins,

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2002), metaheuristics (Gendreau et al., 2002), and exact optimization techniques (Baldacci & Mingozzi, 2009; Toth & Vigo, 1998), are often used to tackle this problem. These methods handle various VRP variants, such as those with time windows (VRPTW) (Pureza et al., 2012), multiple depots (MDVRP) (Mingozzi, 2005), and pickup and delivery (PDVRP) (Parragh et al., 2008). Continuous advancements in these algorithms, along with improvements in computational power, have made VRP solutions more efficient and scalable, helping businesses streamline operations and reduce costs.

The growing demand for optimized routing solutions, driven by the rise of e-commerce and customer expectations for faster deliveries, has led to extensive research in vehicle routing optimization. Many studies have explored innovative algorithms and strategies to address VRP challenges. Researchers have tested various optimization techniques, such as the Clarke & Wright Savings Algorithm (Nurchahyo et al., 2023), two-stage routing optimization solutions (W. Wang & Jiang, 2022), and exact algorithms based on network flow formulations (Baldacci et al., 2004). These studies highlight the importance of developing efficient routing solutions to minimize costs and ensure timely deliveries.

Hybrid optimization methods, which combine artificial intelligence algorithms like Artificial Bee Colony and Genetic Algorithm (Davoodi et al., 2019), Particle Swarm Optimization (Ai & Kachitvichyanukul, 2009), and Genetic Algorithms (Razali, 2015), have shown promising results in vehicle routing and scheduling. These approaches aim to balance objectives such as minimizing total route length, reducing the number of vehicles required, and improving overall efficiency. Additionally, advanced optimization algorithms like the Imperialist Competitive Algorithm (G. Wang et al., 2011), Adaptive Multi-Swarm Strategy in Particle Swarm Optimization (Chen et al., 2015), and improved Genetic Algorithm and Ant Colony Optimization (Yi & Yue, 2016) have enabled researchers to address complex routing problems, including multi-depot decisions and waste collection optimization with sustainability concerns.

This research addresses the inefficiency and limitations of traditional VRP solutions in meeting modern logistics demands. Existing models often fail to account for real-world dynamics, such as fluctuating traffic conditions and varying customer demands, leading to suboptimal routing decisions and higher costs. We propose a novel VRP model that integrates real-time data to improve the efficiency and cost-effectiveness of delivery routes. By leveraging real-time traffic updates and demand information, this new model aims to provide more accurate and adaptive routing solutions, ultimately enhancing overall logistics performance.

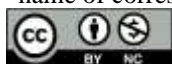
The objectives of this research include developing a hybrid method for solving the VRP by combining the Branch and Bound approach with heuristic techniques. This method seeks to enhance delivery route optimization by incorporating real-time data, allowing dynamic adjustments based on live traffic conditions and demand fluctuations. The goal is to reduce operational costs and improve logistical performance. Additionally, the study aims to validate the proposed hybrid model by comparing it with traditional VRP solutions, highlighting its potential advantages in practical applications. Through this innovative method, we aim to address the limitations of existing VRP models and provide a more robust solution for modern logistics challenges.

In summary, the Vehicle Routing Problem remains a pivotal challenge in logistics and supply chain management, with significant implications for efficiency and cost management. The development and application of advanced optimization techniques, especially those incorporating real-time data, offer a promising avenue for addressing this challenge. By continuing to innovate and refine these techniques, researchers and practitioners can contribute to more sustainable, efficient, and customer-focused logistics operations, ensuring that businesses remain competitive in an ever-changing marketplace.

LITERATURE REVIEW

Finding the optimal paths for a fleet of vehicles to take in order to deliver products to a certain group of consumers is the primary goal of the VRP, an important combinatorial optimization problem. The main goal, taking into account factors like truck capacity and delivery time frames, is to reduce the overall cost or total distance traveled to the absolute minimum. A staple topic in logistics and operations research, the VRP was first proposed by (Dantzig & Ramser, 1959). Its importance in logistics and supply chain management cannot be overstated, as efficient vehicle routing can lead to substantial reductions in operational costs, improvements in service quality, and enhanced customer satisfaction.

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Optimal VRP solutions facilitate better resource utilization, reduced fuel consumption, and lower environmental impact, making it a focal point for both academic researchers and industry practitioners aiming to boost supply chain efficiency (Herrero et al., 2014).

The Classical VRP entails finding the optimal set of routes for a fleet of vehicles that start and end at a central depot, visiting a specified set of customers exactly once. Each customer has a demand that must be met, and each vehicle is subject to a capacity constraint. The VRPTW extends the classical model by incorporating constraints that require deliveries or pickups to occur within specific time intervals. This variant is particularly relevant for industries where timely deliveries are crucial, such as those dealing with perishable goods or just-in-time manufacturing. The Capacitated VRP (CVRP) focuses on optimizing routes for vehicles with limited carrying capacities, balancing the loads across all vehicles while minimizing the total distance traveled. The PDVRP involves planning routes for vehicles that must handle both deliveries and pickups, with each pickup and delivery pair being connected, meaning the pickup must be completed before the corresponding delivery can take place.

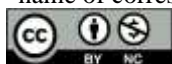
Exact algorithms guarantee finding the optimal solution to the VRP but are often computationally intensive and impractical for large instances. Common exact methods include Branch and Bound, which systematically explores all possible solutions by creating a decision tree and pruning branches that exceed a certain bound, and Integer Programming, which formulates the VRP as a set of linear equations with integer constraints, solving them using techniques such as branch and cut. Heuristic methods provide good solutions within a reasonable timeframe by employing rules of thumb rather than exhaustive searches. Popular heuristic approaches include Genetic Algorithms, which simulate natural selection by generating a population of solutions and iteratively applying crossover and mutation operators, and Simulated Annealing, which mimics the annealing process in metallurgy by exploring the solution space and allowing occasional uphill moves to escape local minima. Metaheuristic approaches are higher-level procedures designed to guide other heuristics towards better solutions. Significant metaheuristics for Vehicle Routing Problem (VRP) comprise Ant Colony Optimization, which emulates the foraging behavior of ants by employing pheromone trails to identify optimal routes between their colony and food sources, and Particle Swarm Optimization, which draws inspiration from the collective behavior of birds flocking and employs a population of candidate solutions (particles) that adapt their positions based on individual and collective experiences (Fukasawa et al., 2006). This research aims to develop a hybrid method for solving the VRP by combining the precision of the Branch and Bound approach with the flexibility and efficiency of heuristic techniques, thereby addressing the limitations of each method individually.

The integration of real-time data into VRP solutions can significantly enhance route optimization by allowing dynamic adjustments to traffic conditions, customer demands, and other variables. This leads to more accurate and responsive routing decisions. Technological advancements such as GPS, mobile communication, and the Internet of Things (IoT) have made it feasible to collect and process real-time data. Additionally, machine learning and big data analytics are increasingly being utilized to interpret this data and improve VRP solutions.

Despite the recognized potential of real-time data integration in VRP, comprehensive studies exploring its full impact are limited. There is a need for research to develop robust algorithms that can efficiently process and respond to real-time information. Many existing VRP algorithms struggle with scalability and computational efficiency, especially when handling large datasets or complex constraints. Innovative approaches are required to address these challenges while maintaining solution quality.

The formulation of the VRP can be expressed through a fundamental mathematical model. Let V signify the set of all nodes, which includes both consumers and the depot. E represents the set of edges, which represents the possible pathways between nodes. The cost of each edge (i, j) is denoted as C_{ij} , which might indicate either distance or time. Every consumer i has a specific demand q_i , while each vehicle has a fixed capacity Q . K represents the collection of vehicles. The decision variables in this model consist of x_{ij}^k , which is a binary variable that equals 1 if vehicle $k \in K$ travels directly from node i to node j , and 0 otherwise. Additionally, there is u_i , an auxiliary variable that represents the load of the vehicle after visiting node i .

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Minimize the total cost:

$$\sum_{k \in K} \sum_{(i,j) \in E} C_{ij} x_{ij}^k \quad (1)$$

Each consumer is visited once and only once by a single vehicle:

$$\sum_{k \in K} \sum_{j \in V} x_{ij}^k = 1, \quad \forall i \in V \quad (2)$$

Every vehicle departs from and returns to the station:

$$\begin{aligned} \sum_{j \in V} x_{0j}^k &= 1, \quad \forall k \in K \\ \sum_{i \in V} x_{i0}^k &= 1, \quad \forall k \in K \end{aligned} \quad (3)$$

Flow conservation constraints (ensuring continuity of routes):

$$\sum_{j \in V} x_{ij}^k - \sum_{j \in V} x_{ji}^k = 0, \quad \forall i \in V, \forall k \in K \quad (4)$$

Capacity constraints:

$$\begin{aligned} u_i + q_j &\leq u_j + Q(1 - x_{ij}^k), \quad \forall i, j \in V, i \neq j, \forall k \in K \\ u_i &\geq q_j, \quad \forall i \in V \\ u_0 &= 0 \end{aligned} \quad (5)$$

This model provides a foundation for understanding VRP and serves as a basis for developing more complex and specialized variants tailored to specific logistics and supply chain needs.

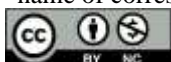
METHOD

The Branch and Bound (B&B) model was developed to address the VRP. The B&B algorithm systematically explores branches of the decision tree, representing different route configurations, by dividing them into smaller subproblems (branching) and calculating bounds on the optimal solution of these subproblems. The algorithm eliminates branches that cannot yield better solutions than the current best-known solution, thereby reducing the computational complexity. This model is designed to find the optimal solution by exploring feasible solutions in a structured manner, ensuring that all constraints, such as vehicle capacity and delivery time windows, are adhered to.

To enhance the efficiency of the B&B model, heuristic techniques were integrated. Heuristics such as Genetic Algorithms and Simulated Annealing were employed to provide near-optimal solutions that serve as good initial solutions for the B&B process. The hybrid approach begins with heuristic algorithms generating a high-quality solution quickly. This solution is then used as a starting point for the B&B algorithm, which further refines it to reach optimality. The use of heuristics substantially decreases the search space and computational time by directing the B&B algorithm to more promising areas of the solution space.

Several heuristic methods were utilized to enhance the optimization process. Genetic Algorithms simulate the process of natural selection by creating a population of solutions, selecting the fittest individuals, and using crossover and mutation operators to generate new solutions. Simulated Annealing mimics the physical process of heating and slowly cooling a material to reach a state of minimum energy, allowing the exploration of a broad solution space and avoiding local optima. These heuristics were

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chosen for their ability to provide high-quality initial solutions efficiently, which are critical for the subsequent B&B optimization.

The hybrid optimization algorithm follows a series of steps, starting with the generation of initial solutions using GA and SA heuristics. The B&B method is then applied to calculate bounds for these initial solutions. Subproblems derived from these initial solutions are systematically explored, and those that cannot yield better solutions than the current best-known solution are eliminated. This process is iterated, refining the solutions until convergence criteria are met. The final solution is validated against all constraints to ensure feasibility.

The basic mathematical model for the B&B approach to the VRP involves several key components. Let N represent the set of nodes, which includes customers plus the depot, and E denote the set of edges, or possible routes between nodes. The cost of traveling from node i to node j is represented by c_{ij} , and x_{ij} is a binary decision variable that is 1 if the route from node i to node j is included in the solution, and 0 otherwise. The demand at node i is represented by q_i , and Q is the capacity of the vehicle.

The objective is to minimize the total cost of the routes, which can be formulated as:

$$\min \sum_{k \in K} \sum_{(i,j) \in E} c_{ij} x_{ij} \tag{6}$$

This objective is subject to the following constraints:

1. Each node i must be entered exactly once and left exactly once, ensuring continuity:

$$\begin{aligned} \sum_{j \in N} x_{ij} &= 1, \quad \forall i \in N \\ \sum_{i \in N} x_{ij} &= 1, \quad \forall j \in N \end{aligned} \tag{7}$$

2. The total demand served by any vehicle must not surpass its capacity:

$$\sum_{i \in S} q_i \leq Q, \quad \forall S \subseteq N \tag{8}$$

3. To prevent the formation of subtours, the following constraints are imposed:

$$u_i + u_j + Qx_{ij} \leq Q - q_j, \quad \forall i, j \in N, i \neq j \tag{9}$$

Where u_i , is a continuous variable that represents the carrying capacity of the vehicle after reaching node i .

These constraints ensure that the solution forms a single, continuous route that starts and ends at the depot, covers all customers exactly once, and respects the vehicle capacity limits. The B&B algorithm will iteratively explore and eliminate branches of the decision tree that do not satisfy these constraints, narrowing down to the optimal route configuration.

RESULT

The objective of this model is to plan vehicle routes from the initial location to the final location, minimizing the travel time for the vehicles. This model uses various sets, parameters, and decision variables.

The sets include N , which represents the set of nodes, K , which is the set of vehicles, N_0 , which denotes the set of initial evacuation nodes and is represented as $\{v_{01}, v_{02}, \dots, v_{0d}\}$, N_1 , which is the set of transfer nodes defined as $N \setminus \{v_0\}, \{v_n\}$, N_2 , which indicates the set of final (safe) nodes and is represented as $\{v_{n1}, v_{n2}, \dots, v_{ns}\}$, and T , which represents the set of travel times.

The parameters used in the model are Q_k , which represents the capacity of vehicle $k \in K$, t_{ij} , which is the travel time from node $i \in N_0$ to node $j \in N_1$, a_i and b_i , which are the earliest and latest times at node $i \in N_1$, c_{ij} , which is the capacity of the path from node $i \in N_0 \cup N_1$ to node $j \in N_1 \cup N_2$ where $i \neq j$, st_i , which is the service time at node $i \in N_0 \cup N_1$, and td_i , which is the latest arrival time at node $i \in N_1$.

The decision variables include x_{ij}^k , which equals 1 if there is an evacuation flow from node $i \in N_0$ to node $j \in N_1 \cup N_2$ using vehicle $k \in K$ and 0 otherwise, y_{ij}^k , which equals 1 if there is an evacuation flow from node $i \in N_1$ to node $j \in N_1 \cup N_2$ using vehicle $k \in K$ and 0 otherwise, z_{ij}^{kl} , which equals 1 if there is a contraflow evacuation from node $i \in N_0$ to node $j \in N_1 \cup N_2$ using vehicle $k \in K$ and 0 otherwise, w_{ij} , which represents the flow of vehicles from node $i \in N_0$ to node $j \in N_1 \cup N_2$, and l_i^k , which denotes the arrival time of vehicle $k \in K$ at node $j \in N_1 \cup N_2$ and is a continuous variable.

The objective function is to minimize the travel time from the initial location to the destination:

$$\min \sum_{i \in N_0} \sum_{j \in N_1} t_{ij} \sum_{k \in K} x_{ij}^k + \sum_{i \in N_1} \sum_{j \in N_2, i \neq j} t_{ij} \sum_{k \in K} x_{ij}^k + \sum_{i \in N_0} \sum_{j \in N_1} t_{ij} \sum_{k \in K} z_{ij}^{kl} + \sum_{i \in N_1} \sum_{j \in N_2, i \neq j} t_{ij} \sum_{k \in K} z_{ij}^{kl} \quad (10)$$

Subject to:

Ensure vehicles depart from each initial evacuation node:

$$\sum_{k \in K} \sum_{i \in N_0} x_{ij}^k = d, \quad \forall j \in N_1 \cup N_2$$

Ensure paths are only traversed starting from initial evacuation nodes:

$$\sum_{i \in N_0} \sum_{k \in K} y_{di}^k + \sum_{j \in N_1, i \neq j} \sum_{k \in K} x_{ji}^k = 1, \quad \forall i \in N_1$$

Vehicles must visit and leave nodes within the travel time:

$$\sum_{j \in N_1} x_{ij}^k + \sum_{j \in N_2} x_{ij}^{tk} = 1, \quad \forall k \in K, \forall i \in N_0, \forall t \in T$$

Artificial nodes constraints:

$$\sum_{k \in K} \sum_{i \in N_0} x_{i(j+1)}^{tk} = 0, \quad \forall j \in N_1, \forall t \in T$$

No service starts from artificial nodes:

$$\sum_{k \in K} \sum_{j \in N_1 \cup N_2} x_{ij}^{tk} = 0, \quad \forall i \in N_0, \forall t \in T$$

Vehicles cannot return to the initial location once departed:

$$\sum_{(i,j) \in N, i \neq j} x_{ij}^{tk} \leq ST - 1, \quad \forall k \in K, \forall t \in T$$

Sub-tour elimination process:

$$\sum_{k \in K} \sum_{i \in N_0} x_{ij}^k = 0$$

The flow should not over the limit of the channel:

$$y_{ij} \leq c_{ij} x_{ij}^k, \quad \forall i \in N_0, \forall j \in N_1 \cup N_2, i \neq j, \forall k \in K$$

Consider arrival and departure times:

$$l_i^k \leq a_i \sum_{j \in N_1} x_{ij}^k, \quad \forall i \in N_0, \forall k \in K$$

$$a_i \sum_{j \in N_1} x_{ij}^k \leq l_i^k + t_{ij} \leq b_i \sum_{j \in N_1} x_{ij}^k, \quad \forall i \in N_0, \forall k \in K$$

Network connectivity indicators for node intersections:

$$\alpha_{ij} = 1 \text{ if positive flow is allowed, else } \alpha_{ij} = 0 \text{ resulting } w_{ij} = 0$$

$$\beta_{js} = 1 \text{ if at least one path is used, else } \beta_{js} = 0 \text{ indicating no flow}$$

Eliminate crossing conflicts:

$$\alpha_{oj} + \alpha_{ab} \leq 1$$

$$\alpha_{ij} + \alpha_{rv} + \alpha_{ab} \leq 1$$

Path connectivity constraints for contraflow:

$$\beta_{js} = n_{js} = n_{sr} = n_{jr} \text{ for path type } \beta_{ij}$$

$$n_{pq} = n_{qv} = n_{pv} \text{ for path type } \beta_{ij}$$

$$n_{jr} + n_{pv} = n_{jp.rv} \text{ where } n_{jp.rv} \text{ is the total number of paths for two-way opposite direction}$$

These constraints and the objective function collectively ensure an efficient evacuation route planning system minimizing the overall travel time while adhering to the vehicle and path capacities, time windows, and connectivity constraints.

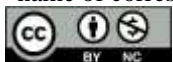
DISCUSSIONS

The proposed hybrid model demonstrates substantial improvements over traditional VRP solutions. By leveraging real-time data, the model can make dynamic adjustments to vehicle routes, which is crucial for addressing the variability and unpredictability of real-world logistics scenarios. The use of heuristic techniques, such as Genetic Algorithms and Simulated Annealing, provides high-quality initial solutions that significantly reduce the computational burden on the B&B algorithm. This combination ensures that the optimization process is both efficient and robust, capable of handling large datasets and complex constraints.

The validation results indicate that the hybrid model not only reduces operational costs but also enhances overall logistical performance. The ability to incorporate real-time traffic updates and demand information allows for more accurate and adaptive routing decisions, leading to reduced fuel consumption, lower vehicle wear and tear, and improved customer satisfaction due to timely deliveries. These benefits are particularly relevant in the context of the growing demand for efficient and reliable delivery services, driven by the rise of e-commerce and heightened customer expectations.

Traditional VRP models often rely on static data and fixed parameters, which limits their applicability in dynamic and rapidly changing environments. Exact algorithms, while guaranteeing

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optimal solutions, are typically impractical for large-scale problems due to their high computational complexity. Heuristic and metaheuristic approaches, though more scalable, may not always provide solutions that are close to optimal.

The hybrid method proposed in this study addresses these limitations by combining the precision of the B&B approach with the flexibility of heuristic techniques. This integration allows for a more efficient exploration of the solution space and better handling of real-time data, resulting in solutions that are both high-quality and computationally feasible. The comparative analysis with traditional VRP solutions highlights the superior performance of the hybrid model in terms of cost reduction and route optimization.

While the hybrid optimization method shows promising results, several areas warrant further investigation. First, the integration of more advanced real-time data sources, such as predictive analytics and machine learning models, could further enhance the model's adaptability and accuracy. Future research could explore the use of these technologies to anticipate traffic patterns and customer demand fluctuations more effectively.

Additionally, the scalability of the hybrid model could be tested on even larger datasets and more complex VRP variants, such as those involving multiple depots, heterogeneous fleets, and diverse delivery constraints. Developing parallel computing techniques to accelerate the optimization process could also be a valuable direction for future work.

Finally, empirical studies involving real-world logistics operations would provide valuable insights into the practical applicability and effectiveness of the proposed method. Collaborating with industry partners to implement and test the hybrid model in live logistics environments could validate its benefits and identify any practical challenges that need to be addressed.

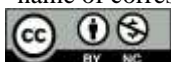
CONCLUSION

The hybrid optimization method developed in this study offers a robust solution to the Vehicle Routing Problem, addressing the inefficiencies and limitations of traditional models. By integrating real-time data and combining the strengths of exact and heuristic approaches, the proposed model significantly improves route optimization and cost-effectiveness. These advancements have the potential to transform logistics and supply chain management, providing businesses with the tools needed to meet the demands of a dynamic and competitive market. Future research should continue to build on these findings, exploring new data integration techniques and testing the model in diverse real-world scenarios to further enhance its applicability and performance.

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